The Calculus of Pythagorean Triples Sequences

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Introduced here is a method for systematically calculating all primitive Pythagorean triples. This is an extension of the work of the Pythagoreans, Plato, Euclid and Fermat. The system arrived at consists of three general triples sequences produced by the search engine Staircase. The triples sequences can be used to produce infinitely many triples tables developed to any depth required. The first 32 tables generated by the individual sequence general terms filled to a depth of 40 triples are included. Six new pairs of incongruent Pythagorean triangles, each pair with the same distinct area, were found in a first search of the 32 tables. Next, primitive triples in the 32 tables were culled and sorted by increasing size to form a sequence of the values of the third largest element r of each triple, r being the hypotenuse of a corresponding Pythagorean triangle. The sequence of values of r is found to consist of either Fermat primes of the form 4 k + 1 or of composites of Fermat primes of the form 4 k + 1 only. Sequences of the primes and composites are listed next the r sequence. The primes' and composites' sequence terms assume one of two versions of the general Euclidian expression for triples, either Euclid's classic expression that coincides with Fermat's expression for Fermat primes or one-half Euclid's expression. Consequently, there are two different but similar expressions for forming each Fermat prime of the form 4 k + 1 as compared to Fermat's original single expression for those primes. Additionally, these two expressions are found to extend to the composites of Fermat's primes only. A further transformation into 16 Fermat systematic term sequences representing an inherently natural reorganization of the r sequence into the sequence of primes, sequences of powers of primes and sequences of composites of primes are created. The search algorithm Latchkey was designed to find the primitive triples corresponding to any Fermat prime of the form 4 k + 1 or any composite of only these primes. The number of primitive triples pairs it finds for a particular composite increases exponentially with the number of distinct Fermat primes in the composite. The detection of Fermat primes and their composites by Latchkey renders them insecure in cryptography. Primitive triples sequences corresponding to fixed difference of the orthogonal sides of the Pythagorean triangles were determined. The algorithm Fixed |y – x| Primitive Triples was designed to find the infinite sequences of Pythagorean triples with a fixed value |y – x|. Parallel recurrence sequences were discovered which were necessary to extend the fixed |y – x| triples sequences.

A. Introduction

Apostol in his book *Introduction to Analytic Number Theory* offers a prefatory essay on the history of number theory[1]. Apostol describes the two historic methods for the calculation of Pythagorean triples, both known in antiquity. The first method satisfies the condition that the length of hypotenuse r less the length of the long orthogonal side y of a Pythagorean triangle is 1. This procedure is attributed to the Pythagoreans[2]. A second method with the incremented condition r – y = 2 is attributed to Plato[2]. The notations for the orthogonal sides, x and y are conventional notations for triples' elements, but r representing the hypotenuse of the corresponding right triangle is not conventional, a choice explained below. Euclid found a general expression that produced triples[3] but had no systematic way to exploit it. A systematic method for finding all triples is developed below expanding on the work of the Pythagoreans and of Plato. Euclid's expression for triples and Fermat's sum of two natural number squares[4] to form Fermat primes of the form 4 k + 1 both emerge naturally from the sequence general terms developed in the work of the Pythagoreans, of Plato and of the new triples sequences reported below.

As described in Sec. B, the two systematic procedures discussed by Apostol are expressed algebraically facilitating the calculation of x, y and r, methods unknown to the ancients. The general terms for the calculation rules for each component of the triples for both procedures are stated in terms of a variable n, a standard notation where n takes values from the natural numbers. Also in Sec. B it is shown that the calculation rules can be expressed as infinite sequences of triples components. The two triples calculation methods mentioned above are the first such for the systematic computing of triples, two of an infinite number of such procedures. The rest of these developed in Sec. C.2 are new expressions beyond the two classic procedures.

Introduced in Sec. C.1 is the search algorithm Staircase. The output of Staircase enables searches for the difference of natural number squares that are also natural number squares. Staircase calculates trial triples in tabular columns starting with a grouping of columns labeled Step 1 in Table SA. The next triples calculations appear in Table SA one row down in a grouping of columns directly to the right, labeled Step 2. Each following consecutive calculation procedure is another step down to the right of the previous one and so on in a stair step pattern. The pattern by design is Step 1 with $r - y = 1$, Step 2 with $r - y = 2$, Step 3 with $r - y = 3$ and so on. Staircase in principle can compute all possible Pythagorean triples by computing all possible differences of r^2 and y^2 constrained by $r - y$ being a natural number systematically taken from the sequence of natural numbers for each new step. Staircase should reproduce the Pythagoreans' triples list in Step 1 and Plato's triples list in Step 2 and it does.

The results found from employing Staircase for 16 steps beyond Step 2 were searched by inspection for patterns of new triples among the step entries in Table SA. Step 8, Step 9 and Step 18 each yielded new triples different from those in Steps 1 and 2 and different from each other. The triples found in each of the other steps were multiples of triples found in the step columns with smaller step numbers than the column being considered. The search for new Pythagorean triples for steps 8, 9 and18 adds three distinct new triples expressions in addition to those from steps 1 and 2. At this point there were five distinct triples calculation rules for five steps: steps 1, 2, 8, 9 and 18. This was sufficient to determine three distinct general triples calculation rules that systemize the calculation of all possible triples. These new general rules produce both primitive triples and as well as some non-primitive triples.

32 tables of triples sequences, labeled Table 1 through Table 32, were computed from the three general infinite triples sequences. Half of the tables, those with odd table

numbers 1 through 31, have two table components. Even numbered tables have one table component. It is proved that tables with table number 1 and table numbers 2^k, where k is the sequence of natural numbers, contain primitive triples only. All odd numbered tables with table numbers ≥ 3 and all even table numbers outside of those with table numbers 2^k have one or more non-primitive triples sequences, these occurring in regular patterns. The production of non-primitive triples patterns is analyzed in Sec. D.

In Sec. E a two-stage reorganization of the general triples sequences is described. The first stage of this reorganization places the values of r of the three general triples sequences together into a single sequence of ascending order in Table RPC1. In this initial stage of reorganization the first 344 consecutive values of r belonging to primitive triples only are taken from Tables 1 – 32. r values of non-primitive triples were excluded due to the redundancy of non-primitive triples. The sequence of r values is then sorted into two additional separate sequences in Table RPC1: the sequence of primes and the sequence of composites of primes. The sequence of primes consists of Fermat primes of the form 4 k + 1 only and sequence of composites are composites of Fermat primes of the form 4 k + 1 only. Fermat had discovered that primes of the form 4 k + 1 can be written as the sum of the squares of natural numbers[4]. Developed in Sec. E.3 is that composites of only Fermat primes also follow Fermat's sum of squares rule. Also developed there is that Fermat primes and composites of Fermat primes are generated by a new rule: one-half the sum of the squares of natural numbers. These developments emerge from the analysis and discussion of Table RPC2.

The second stage of reorganization of the 32 tables is the rearrangement of the terms of sequences in Table RPC1 by regrouping the terms of those sequences into systematic terms sequences whose general terms can be expressed as a function of the natural numbers ν as required for an infinite sequence general term. This new arrangement is found in Table FSTS. Table FSTS provides new patterns and fresh perspectives on the generation of Pythagorean triples.

Introduced in Sec. E.6 is search algorithm Latchkey. This search algorithm was developed to find all triples corresponding to a particular value of r where r is a Fermat prime or a composite formed from only Fermat primes. Latchkey is similar in action to Staircase in that they both employ the Pythagorean theorem as a basis of their action. From Latchkey it is found that increasing the number of distinct Fermat prime factors in a composite of only Fermat primes results in an exponential increase in the number of primitive triples corresponding to the particular value of r produced by the general triples sequences. Latchkey can thus detect Fermat primes and also the number of distinct Fermat primes in a Fermat composite narrowing the factorization procedure for such composites.

In Sec. F, triples sequences with fixed $|y - x| = \Delta$ are found where Δ takes values 1, 7, 17 and 23. These values of Δ are the first four of an infinite sequence of such values generated by the general triples sequences. Such triples sequences were found using the discovered parallel recurrence sequences in combination with the algorithm Fixed $|y - x|$ Primitive Triples.

B. Euclidean and Cartesian Geometry of Pythagorean Triples

The Pythagorean theorem, Pythagorean triangles and Pythagorean triples were known to the ancients. A Pythagorean triple as presented here is the ordered three-member list of the natural number lengths of the sides of a Pythagorean triangle with the hypotenuse being the last, largest member of the list. The notation for triples adopted here is a list of the length of the three sides enclosed by braces: $\{x, y, r\}$ where x, y, and r are natural numbers. The symbol r is chosen to represent the hypotenuse since points (x, y) and (y ,x) in the Cartesian plane will be the natural number solution points on the graph of the equation $x^2 + y^2 = r^2$ in the first quadrant sector of the circle of radius r. Coordinate axes – circle intersection points that require either x or y be 0 are excluded since 0 is not a natural number.

The first Fermat prime of the form $4\ k + 1$ occurs when $k = 1$ where $4 \cdot 1 + 1 = 5$. The first quadrant sector of the circle with $r = 5$ is plotted in Fig. 1. The natural number solution points for that circle are the points (3, 4) and (4, 3). The Pythagorean triples corresponding to the triangles in Fig. 1 are {3, 4, 5} and {4, 3, 5}. The convention for ordering the orthogonal sides of a Pythagorean triangle, adopted here and illustrated in Fig. 1, is to proceed counter-clockwise from origin around the triangle starting with the value of x on the x-axis. The ordering of the triples follows that of the triangles. The two triangles corresponding to triples {3, 4, 5} and {4, 3, 5} in Fig. 1 are congruent.

Primitive triples play a dominant role in the developments below. A primitive triple is a triple that has three elements that have no common factors greater than 1. The examples in Fig. 1 are primitive triples. It will be proved that for primitive triples, x and y are either x even and y odd or *visa versa* requiring that r is always odd. Symbols assigned for particular values of x and of y for primitive triples are $x = a$ and $y = b$ where a and b are natural numbers. The corresponding natural number solution points on the circle of natural number radius r are (a, b) and (b, a) for those values of r that have natural number solution points. For $r = 25$, where $k = 6$ and $4\ k + 1 = 25$, (a, b) = (7, 24) and (b, a) = (24, 7). 25 is a power of the Fermat prime 5 where $r - y = 1$ for point (7, 24), a Step 1 triple.

Olmsted[5] defines an infinite sequence as a single-valued function with domain of definition being the natural numbers. The n*th* term of sequence, called the general term, is usually an expression with n as the independent variable. In the considerations below ν is reserved to represent the domain of the sequence of natural numbers. However n is the traditional variable used in the expression for triples. As will be seen n is a linear function of ν with n(ν) being either even or odd. ν, the actual domain variable of the sequences developed below, is in plain sight serving as row number in all triples sequences tables.

The first Pythagorean triple lists[2] as described by Apostol is the set of three lists where n is the independent variable in each expression for each component in the triple:

$$\{x(n), y(n), r(n)\} = \{n, \tfrac{1}{2} (n^2 - 1), \tfrac{1}{2} (n^2 + 1)\}. \tag{1}$$

Apostal has n running through the *odd* natural numbers starting at 3 ending at 19. As mentioned above the variable n in Eq. 1 does not satisfy the definition of the independent variable for an infinite sequence, this being taken up directly below. That condition $r - y = 1$ applies as can be demonstrated generally by subtracting y(n) from r(n), both functions appearing in Eq. 1. The evaluation of the three lists x(n), y(n) and r(n) using odd n from 3 to 81 is found in Table 1.1 where $r - y = 1$ in each row. Note that the row numbers are ν. The table number is 1; "." means "part" and ".1" means "part 1".

The Plato triples list[2] described by Apostol takes the following form: $\{x(n'), y(n'), r(n')\} = \{4 n', (4 (n')^2 - 1), (4 (n')^2 + 1)\}$ where n′ is the natural numbers beginning at 2 and ending at 5, a pattern quite different from Eq. 1. The Plato triples lists transformed by the change to variable $n = 2 n'$ results in the new pattern $\{2 n, n^2 - 1, n^2 + 1\}$. This new pattern for the Plato triples lists is 2 times Eq. 1 and is adopted from this point forward as

$$\{x(n), y(n), r(n)\} = \{2 n, n^2 - 1, n^2 + 1\} \tag{2}$$

n takes the even numbers starting at 2. This transformation was discovered after development of Staircase and the General Triples Sequences. That the original version of Plato's lists reported by Apostol follows a different pattern from Eq. 1 may have discouraged further search for new lists for Pythagorean triples.

The evaluation of Eq. 2 for even n from 2 to 80 is found in Table 1.2 where the condition $r(n) - y(n) = 2$ holds. Recall that ν is both row number and independent sequence variable. The numerical factor for both y(n) and r(n) in the right side expression in Eq. 2 is 1 in contrast to the ½ in the right side expression in Eq. 1. Eq. 2 is an essential element in finding the three general triples sequences described in Sec. C.2. That $y = r - 1$ for Eq. 1 and $y = r - 2$ for Eq. 2 suggests a pattern for general triples formulations.

Apostol correctly does not characterize x(n), y(n) or r(n) as general terms of infinite sequences. In order that x(n), y(n) and r(n) in Eq. 1 and Eq. 2 to perform as though general terms of a sequence, linear relationships between n and ν, the domain of definition, must be provided. For the Pythagorean triples sequences, $n(\nu) = 2\nu + 1$ relates the domain of natural numbers ν to the traditional form using variable n which in turn determines the values of x(n), y(n) and r(n) in their respective infinite sequences. For the Plato triples sequences, $n(\nu) = 2\nu$ relates the domain of natural numbers ν to the traditional variable n. Substitution of these linear relations into the traditional general expressions of the triples of Eq. 1 and Eq. 2 results in non-intuitive expressions in ν and so is avoided throughout this work. This choice also simplifies the analysis of non-primitive triples in Sec. D. Furthermore retaining n in the sequence formulations of Eq. 1, Eq. 2 and all new triple formulations is crucial in linking Fermat primes of the form $4k + 1$ to primitive triples as developed in Sec. E. The relation $n(\nu) = 2\nu + 1$ starts the Pythagorean triples sequences off at $n(1) = 3$ and the relation $n(\nu) = 2\nu$ starts Plato triples sequences off at $n(1) = 2$. General methods for determining $n(\nu)$ are developed in Sec. C.2.

The first triple produced by Plato triples sequences in the new formulation of Eq. 2 is {4, 3, 5} where x and y of the Pythagorean triple {3, 4, 5} have been exchanged. This first triple picks up the natural number solution point (4, 3) for the circle with $r = 5$ in Fig. 1 where $x(2) > y(2)$ in Table 1.2. The value $n' = 1$ in Apostol's version Eq. 2 may have been avoided historically since it produces triple {4, 3, 5} which was perhaps taken to be an out of order, redundant triple. The idea argued here is that triples come in pairs because natural number solution points come in pairs. Triples with $x(n) > y(n)$ occur in all tables except Table 1.1 where $x(n) < y(n)$ in all rows of that table. Furthermore Table 1.1 cannot produce multiples of any of its triples since such a triple would require $r - y > 1$ in contradiction to the required condition $r - y = 1$ for Table 1.1. (x, y, and r would all have to have the same factor of 2, or 3, etc.). Thus Table 1.1 produces no non-primitive triples. But as developed below, triples of the form {x, y, r} where $x(n) > y(n)$ are produced by all new triples sequences found beyond Table 1.1 by Staircase and by necessity the new general triples sequences developed below. Further developments in Sec. E.3 require the inclusion of primitive triples with $x(n) > y(n)$ that compliment triples with $x(n) < y(n)$.

Fermat's primes of the form $4k + 1$ are primes in the infinite sequence 5, 9, 13, 17, 21, and so on. This sequence consists of every other odd number starting at 5. The primes in this sequence are 5, 13, 17, 29, …. Sec. E.1 has an extended computation of this sequence. Fermat discovered that these primes could be computed by the sum of squares of natural numbers. This result follows naturally from the general triples sequences described in Sec. C. 2 below. The general sequence term for r in Eq. 2 is $r(n) = n^2 + 1^2$. From Table 1.2, $n = 2$ for triple {4, 3, 5} so that $r(2) = 2^2 + 1^2 = 5$. The general term for r in Eq.(1) is $r(n) = ½(n^2 +1)$. From Table 1.1, $n = 3$ for triple {3, 4, 5} and $r(3) = ½(3^2 + 1^2) = 5$. Thus there are two different ways to determine the Fermat primes of the form $4k + 1$ for a given value of k that follows from the evaluation of r(n) in all new general triples sequences. The same result also obtains for composites of Fermat primes of the form $4k + 1$. These considerations will be returned to and expanded on below, especially in Sec. E.3.

The family of curves $x^k + y^k = r^k$ with natural number $k \geq 2$ are symmetric with respect to line $y = x$. k is traditional here and should not be confused with the index k of Fermat's primes. Several of these curves, each with $r = 5$, are plotted in Fig. 1. The values of k are marked next to each curve. If a photocopy of Fig. 1 were folded along line $y = x$ and held to light, the upper part of any curve in Fig. 1 above $y = x$ would coincide with that curve's locus below $y = x$. Consider points (x, y) and (y, x) on any curve in Fig. 1. Construct a line through points (x, y) and (y, x). This line will be bisected by and perpendicular to line $y = x$. Thus the two points (x, y) and (y, x) are mirror images of one another with respect to line $y = x$ which plays the role of a plane mirror.

The sketch of the general case for reflection symmetry with respect to line $y = x$ is developed in Fig. 2 for the circle ($k = 2$). Constructions on Fig. 2 are the following:

1. the first quadrant sector of the circle of radius R
2. line $y = Y_2$ through point U and line $y = Y_1$ through point L
3. line $x = X_2$ through point L and line $x = X_1$ through point U
4. line $y = x$ and line $y = -x + c$, this second line through points U and L
5. directed line segments $OU = R_U$ and $OL = R_L$ (position vectors or polar vectors)

Line $y = Y_2$, line $x = X_2$ and the coordinate axes form rectangle OX_2SY_2 , line $y = Y_1$, line $x = X_2$, line $y = Y_2$ and line $x = X_1$ form rectangle DLSU, and the x-axis, Line $x = X_1$, line $y = Y_1$, and the y-axis form rectangle OX_1DY_1. The diagonals of these rectangles are line segments on line $y = x$ implying that all three rectangles are squares. Line $y = -x + c$ contains the line segment UL. The coordinates of point U are (x_U, y_U) which makes the constant c in $y = -x + c$ to be $c = x_U + y_U$. Consequently

$$x_L = OX_2 = OY_2 = X_1U = y_U$$
$$y_L = X_2L = X_1D = OX_1 = x_U$$

so that the coordinates of point L are (y_U , x_U).

The line $y = -x + c$ can be moved about on Fig. 2 by adjusting c between bounds of the line passing through circle intercepts (0, R) and (R, 0) where $y = -x + R$ and point T where it would coincide with the tangent line shown there through T. Triangles DCU and DLC are congruent 45° isosceles triangles so that DC = UC = CL. Thus U and L are equal distances from line $y = x$. $y = x$ is perpendicular to $y = -x + c$. (x_U, y_U) and (x_L, y_L) are optics-like images relative to line $y = x$. These results also apply to any member of the family of curves $x^k + y^k = r^k$ with $k > 2$ where, unlike the circle, the symmetry stands out. However marking the natural number solution points on the circle imprints the symmetry on the circle.

From here forward, reflection symmetric points about line $y = x$ are called image points. The image points of interest are natural number solution points on circles of radius r, (a, b) and (b, a), points from now on called Pythagorean image points (Pips). The Pips considered above are points (3, 4) and (4, 3) in Fig. 1. Pip (3, 4) lies above line $y = x$ and is labeled U for upper; Pip (4, 3) is below line $y = x$ and is labeled L for lower. The Pythagorean triples {a, b, r} are called upper triples and the triples {b, a, r} are called lower triples. In Fig. 2 line $y = -x + c$ can be fitted to the Pips where $x_U = a$, and $y_U = b$ resulting in line $y = -x + a + b$. Curves with $k > 2$ have no natural number solution points as per Fermat's Last Theorem. There are cases considered below where there are values of r that correspond to more than one pair of Pips and thus more than one pair of triples corresponding to a given circle. In that case U and L will have identical subscripts for a particular Pip pair. Fig. 3 shows an example of two pairs of Pips corresponding to two pairs of primitive triples on the circle of radius 65. Fig. 4 shows additional examples of more than one pair of Pips. Table LS4 documents the case where there are 16 pairs of Pips corresponding to 16 pairs of primitive triples on the circle of radius 1,185,665, a Fermat composite of the form 4 k + 1.

Shown also in Fig. 1 are the two congruent triangles corresponding to the triples pair {3, 4, 5} and {4, 3, 5}. The Pythagorean triangle with vertex at (3, 4) is a 3-4-5 triangle and the triangle with vertex at (4, 3) is a 4-3-5 triangle. r, anchored at the origin, can be construed as a vector pointing from the origin to one of the two vertices of the two triangles as in Fig. 1 and Fig. 2.

Primitive triples have a greatest common divisor α of the elements of the triple of α = GCD(a, b, r) = 1. Whole number multiples of 2 or greater times a primitive triple is called non-primitive triples where $\alpha > 1$. The primitive triple factor of the non-primitive triple is abbreviated as the PT-factor. Likewise the Pip of a non-primitive triple is α = GCD(a, b, r) times its PT-factor's Pips. α can be even or odd, but α for non-primitive triples produced by the general triples sequences (abbreviated GTS) will be shown to be odd only. However Staircase considered below yields non-primitive triples with α being either even or odd. All GTS tables including Table 1.1 and Table 1.2 have a column labeled α giving the value of α for each triple in the table. The appearance of non-primitive triples is signaled by odd $\alpha > 1$ beginning in Tables 3.1 and 3.2. However Staircase begins producing non-primitive triples with $\alpha = 2$ in Step 2 as shown in Table SA.

The sequence ν is 1, 2, 3, and so on. The product of the Pythagorean triple {3, 4, 5} and sequence ν is the sequence of triples {3, 4, 5}, {6, 8, 10}, {9, 12, 15}, {12, 16, 20}, {15, 20, 25}, and so on. The product of sequence ν and polar coordinates (r, θ) of triple {3, 4, 5} is the sequence (5, 53.13°), (10, 53.13°), (15, 53.13°), (20, 53.13°), (25, 53.13°), and so on. Only the first term of either sequence is a primitive triple. (r = 25 also corresponds to the upper primitive triple {7, 24, 25} where θ is about 73.7°). The angle coordinate of the polar coordinates is the same for all elements of the second sequence above. This implies that a non-primitive triple has the same rational point (x/r, y/r) as the primitive triple to which it corresponds. This can be observed in the triples tables, Tables 1 – 32. For example row 15 in Table 15.1 has rational point (0.6, 0.8); the PT- factor with this rational point is found in the first row in Table 1.1. For a non-primitive triple {x, y, r} = a {x/a, y/a, z/a} where {x/a, y/a, z/a} is the PT-factor residing in a table with a smaller table number. The simplest fundamental representation of a non-primitive triple is its PT-factor. The PT-factor is the "atom" of triples. In the GTS tables all non-primitive triples produced by the general triples sequences are provided with a reference column s_p (where subscript p stands for primitive) which gives the step number of the corresponding PT-factor. s_p is calculated by *Excel* from information in the GTS tables as considered in Sec. C.3.

C. General Pythagorean Triples Sequences

Defined in this section is the search algorithm called Staircase. Staircase computes trial solutions of the Pythagorean theorem in the search for natural number solutions for the length of side x of a Pythagorean triangle as introduced in Sec. B. Three new triples sequences were discovered in the first 18 steps of Staircase. The new triples sequences along with the two classic triples sequences considered above in Sec. B were then generalized to form three comprehensive General Triples Sequences: TS1, TS2 and TS3.

C.1 Staircase in Action

Table SA contains steps 1, 2 and 3 in Staircase and then skips ahead to step 8, concluding there. Accordingly Table SA is a very abbreviated version of the output of the larger computational program used to accumulate the information reported later in this section. The program steps appear as the steps in a staircase as the program was deliberately constructed, the steps going downward toward the right in Table SA. This method of the programing structure provides an intuitive aid in finding the elements of triples. Further, the step number $s = r - y$ figures prominently in the programing procedure for Staircase and is used throughout this paper.

By definition the step number of each step of the program is distinct. Half of the tables, those with odd table numbers, have two table components requiring that there be two distinct values of s for a given odd table number. Tables with even table numbers have one component with one distinct value of s where these values of s differ from those belonging to the odd table numbers. The following description of the construction and functioning of Staircase makes frequent reference to Table SA. The programming language used to create the data summarized in Table SA is *Excel*.

It was noticed for Table 1.1 that $r - y = 1$ and for Table 1.2 that $r - y = 2$. This observation suggests that the pattern $s = r - y$, where s is a member of the sequence of natural numbers. In this way s can provide trial values of y using the definition of s. Then the trial y can be entered into the Pythagorean Theorem to produce a trial value of x.

Staircase is defined as follows. An arithmetic progression is propagated down column *ν* in Table SA producing the sequence of natural numbers in that column. These entries also serve as row numbers and are also seed numbers for the next column, column r^2 where the built-in *Excel* square function was used to propagate r^2 down the column. Since column r^2 for each step is seeded by column *ν*, ν can be interpreted as r when appropriate. On the other hand ν will also provide a value for y as the development of the definition of Staircase below will demonstrate. After the creation of column r^2, the program is transformed into staircase form. Program step 1, so labeled in Table SA, is formulated by moving the next instruction set down one row from row 1 to row 2 for the next three columns. By design Staircase captures every Pythagorean Triple.

For step 1 in row 2 of Table SA r is 2 and by construction of Table SA s is 1. Column x^2 Excel is programed to solve s = r – y for y and solve $x^2 + y^2 = r^2$ for x^2. Excel finds y = 1 (not reported) and $x^2 = 3$ as reported in column x^2. Thus row 2 column x cell in Table SA is the square root of 3, not a natural number. So there is no triple in row 2. However, as column x^2 and column x are propagated down the table using standard *Excel* procedures, natural numbers in column x appear in rows 5, 13, 25 and 41 where the r values are 5,13, 25 and 41 from column ν and the y values are one row up in column ν via s = 1 = r – y so that y = r – 1 (In general y will be s rows up). These are the first 4 successful trial calculations in step 1 and for Staircase. More successes would follow if Table SA were extended farther downward. Staircase as defined above produces the triples in step 1 of Table SA that match those in Table 1.1 created by evaluating Eq. 1. In Table SA the x values are the odd numbers 3 through 9 just as in Table 1.1. Step 1 is the only step for which Staircase produces no non-primitive triples as will be noticed below.

Step 2 is programmed the same way with the continuation of the pattern s = r – y = 2. As can be seen in Table SA the program produces natural number values of x. At step 2 the first triple is the symmetric triple {4, 3, 5} corresponding to {3, 4, 5} from step 1 in Table SA and row 1 in Table 1.2. Step 2 is also the first step in the Staircase algorithm that produces non-primitive triples where they alternate with primitive triples. The first non-primitive triple is {6, 8, 10} = 2 {3, 4, 5}. Directly below each non-primitive entry is the PT-factor in **bold** *italics* calculated by dividing the non-primitive triple by α. All step 2 non-primitive triples are twice PT-factors in Table 1.1. The program for step 2 filled down for the values of ν produce natural number values of x for primitive triples in rows 5, 17 and 37. The first 40 triples calculated using Eq. 2 are in Table 1.2.

Steps 3 through 18 were programmed similarly. This procedure can be extended in principle as far as desired. However the spacing between the natural number values of x will become progressively larger and the distance between steps with new triples sequences will become progressively larger (the step number as developed below is proportional to m^2) making Staircase progressively unwieldy as the step number increases. A more sophisticated program could report only successful trial calculations in the output of the program, but it was deemed valuable to see the false trials and to employ an intuitive program. As it happened row values through 18 were just enough to puzzle out the patterns of the general triples sequences reported below.

Step 3, step 5 and step 7 produce only non-primitive triples that have values of α of 3, 5, and 7 respectively with PT-factors from the Pythagorean triples in Table 1.1. Step 4 produces alternating PT-factors from step 2 with α = 2 and from Step 1 with α = 4. Step 6 produces alternating PT-factors with α = 3 for step 2 PT-factors and α = 6 for step 1 PT-factors.

At step 8 in Table SA new triples appear. Step 8 produces new primitive triples as well as non-primitive triples intermixed similar to step 2. The first natural number trial root of the difference of squares is x = 12 in row 13 so that r = 13. s = 8 = r – y yields y = 5 for r = 13. The primitive triple is {12, 5, 13} which is the symmetrical with {5, 12, 13} in step

1 row 13. {12, 5, 13} is a new primitive triple, so that Step 8 produces the first of the new triples sequences following step 2. The next triple is {16, 12, 20}, a non-primitive triple with $\alpha = 4$ and PT-factor {4, 3, 5} from Table 1.2. The next natural number root is x = 20 in row 29 producing the primitive triple {20, 21, 29}, a classic triple (with y – x = 1). The next triple is {24, 32, 40}, a non-primitive triple with $\alpha = 8$ and PT-factor from Table 1.1.

All steps beyond step 1 the Staircase produces non-primitive triples so all steps are productive in this sense. So Staircase is the first source of non-primitive triples. A second source of non-primitive triples is a significant fraction of the new GTS, TS1, TS2 and TS3, as developed below. The understanding of the occurrence of these non-primitive triples is developed in Sec. D. Demonstrated above is how Staircase was created and used. What follows next is a report of the primitive triples that were obtained from Staircase for steps 8, 9 and 18.

The primitive triples found in step 8 are {12, 5, 13}, {20, 21, 29}, {28, 45, 53}, {36, 77, 85}, {44, 117, 125}, {52, 165, 173}, {60, 221, 229} and {68, 285, 293} where the step number s = r – y = 8. The step 8 triples sequences were determined by pattern searching with comparison to Eq. 1 and Eq. 2: {x(n), y(n), r(n)} = $\{4\,n, n^2 - 4, n^2 + 4\}$ with n odd beginning at n = 3. GTS Table 2 entries were computed using this triples sequence. In Step 8 in Table SA there are non-primitive triples alternating with each primitive triple. The evaluation of Step 8 triples using the relation determined just above exhibits no non-primitive triples in Table 2. Consequently the non-primitive triples produced in Step 8 in Table SA are products of Staircase alone.

Step 9 produced the following primitive triples interspersed with non-primitive triples: {15, 8, 17}, {21, 20, 29}, {27, 36, 45}, {33, 56, 65}, {39, 80, 89}, {45, 108, 117}, {51, 140, 149}, {57, 176, 185}, {63, 216, 225}, {69, 260, 269} and {75, 308, 317} where the step number is s = r – y = 9. The non-primitive triples were included in this list since they are produced by the step 9 GTS. This result is duplicated by some fraction of the GTS sequences determined below. The step 9 GTS with both primitive triples and non-primitive triples were determined by pattern searching with comparison to Eq. 1, Eq. 2 and the GTS sequence determined in step 8 above. The result is {x(n), y(n), r(n)} = $\{3\,n, \tfrac{1}{2}(n^2 - 9), \tfrac{1}{2}(n^2 + 9)\}$ with n odd beginning at n = 5. The new triples sequences were used to compute the evaluations in Table 3.1. Step 9 triples sequences are similar to the step 1 triples sequences, but unlike step 1, every third triple in Table 3.1 is a non-primitive triple. This regularity of the non-primitive triples is considered in Sec. D. The non-primitive triples are multiples of PT-factors from Table 1.1 in Table 3.1, column s_p.

Step 18 produced the following primitive triples interspersed with non-primitive triples every third triple: {24, 7, 25}, {36, 27, 45}, {48, 55, 73}, {60, 91, 109}, {72, 135, 153}, {84, 187, 205}, {96, 247, 265}, {108, 315, 333}, {120, 391, 409}, and {132, 475, 493} where the step number is s = r – y = 18. As above the new triples sequences were determined by pattern searching with comparison to Eq. 1, Eq. 2, step 8 and step 9 triples sequences. The result is {x(n), y(n), r(n)} = $\{6\,n, n^2 - 9, n^2 + 9\}$ with n even starting at n = 4. These sequences have similarities with both the Step 8 and Step 9 triples sequences. The sequences evaluations can be found in Table 3.2. The non-primitive triples produced by the Step 18 triples sequences are multiples of PT-factors from Table 1.2 via column s_p in Table 3.2.

The triples sequences found above are listed for comparison and reference:

Step 1:	$\frac{1}{2} \cdot \{2 \cdot 1\ n,\ n^2 - 1^2,\ n^2 + 1^2\}$, n odd, $n \geq 3$
Step 2:	$\{2 \cdot 1\ n,\ n^2 - 1^2,\ n^2 + 1^2\}$, n even, $n \geq 2$
Step 8:	$\{2 \cdot 2\ n,\ n^2 - 2^2,\ n^2 + 2^2\}$, n odd, $n \geq 3$
Step 9:	$\frac{1}{2} \cdot \{2 \cdot 3\ n,\ n^2 - 3^2,\ n^2 + 3^2\}$, n odd, $n \geq 5$
Step 18:	$\{2 \cdot 3\ n,\ n^2 - 3^2,\ n^2 + 3^2\}$, n even, $n \geq 4$

With these results the consideration of Staircase comes to a close. However an important legacy of Staircase is the step number s as evidenced directly above. Step numbers have a roll in the considerations below in the development of the general triples sequences.

C.2 General Triples Sequences TS1, TS2 and TS3

The expressions in the five triples sequences in the list of five steps, Step 1 through step 18 above, were factored by design to emphasize several patterns. ½ has been factored out of step 1 braces and step 9 braces. The factor of 2 in the coefficient of n in the first element in all braces has been emphasized as a product. Comparing the individual elements in each of the 5 braces shows a pattern of natural numbers 1, 1, 2, 3, 3 in the first element and those numbers squared in the second and third elements in all the triples. Substituting the variable m for this cycle of natural numbers in all of the 5 triples sequences results in general expressions in all braces becoming $\{2\ m\ n,\ n^2 - m^2,\ n^2 + m^2\}$. As argued below, m will be a natural choice for GTS table number where odd m will have two GTS and even m one GTS. And the general expressions in the braces are immediately recognizable. They appear in Euclid's work on Pythagorean triples[3], in popularizations of number theory, and in textbooks such as Apostal's[3]. Furthermore the third element in the braces is the sum of the squares on natural numbers which Fermat discovered to be generators of Fermat primes of the form 4 k + 1. It is the r sequences that generate these Fermat primes. This idea will be explored and extended in Sec. E.3. Step 1 and step 2 combined with step 9 and step 18 suggest that odd m have two independent components requiring two independent table components whereas even m has one component. Referring back to the list of step 1 through step 18 above, there are three kinds of general triples. Step 1 and step 9 have the general expressions in the braces multiplied by ½ where m is odd with m = 1 for step 1 and m = 3 for step 9 and where n is odd. Step 2 and step 18 have the general expressions in the braces multiplied by 1 where m is odd at 1 for step 1 and 3 for step 9 and n is even. Step 8 has the general expressions in the braces multiplied by 1 where m is even starting at 2 and n is odd. If the pattern continued, the next step after step 18 should have m = 4 and n odd similar to step 8 completing the second 3 member cycle of triples sequences: steps 1 and 2, step 8, and steps 9 and 18 with the next step having m = 4. This is exactly what is found in the next table after Table 3.2: Table 4 completes the second 3 member cycle pattern of triples sequences considered at the end of Sec. 3.1 above.

The three GTS (general triples sequences) formulated from the first five triples sequences modified according to the substitutions proposed above are listed below. A nested function notation was developed for the new GTS: $TSj(s_j(m), n)$. j runs from 1 to 3. A casual inspection of all 32 tables shows that the entries of r and y in each table differ by the same step number $s_j(m)$ that uniquely identifies each GTS. The list of the three GTS is:

TS1(s(m), n): $TS1(s_1(m), n) = ½ \cdot \{2\ m\ n, n^2 – m^2, n^2 + m^2\}$, m odd, $m \geq 1$, n odd, $n \geq n_1$, and $s_1(m) = r(m, n) – y(m, n) = m^2$

TS2(s(m), n): $TS2(s_2(m), n) = \{2\ m\ n, n^2 – m^2, n^2 + m^2\}$, m even, $m \geq 2$, n odd, $n \geq n_2$ and $s_2(m) = r(m, n) – y(m, n) = 2\ m^2$

TS3(s(m), n): $TS3(s_3(m), n) = \{2\ m\ n, n^2 – m^2, n^2 + m^2\}$, m odd, $m \geq 1$, n even, $n \geq n_3$ and $s_3(m) = r(m, n) – y(m, n) = 2\ m^2$

The list of the five first triples sequences at the end of Sec. C.1 is: TS1(1, n), TS3(2, n), TS2(8, n), TS1(9,n) and TS3(18, n). The next member of this list should have m = 4: the triples sequences TS2(32, n) at step 32 results forming Table 4. Table 4 contains all new, all primitive Pythagorean triples with m = 4 successfully implementing the sixth GTS to be added to the list of five triples sequences listed at the end of Sec. C.1. The rest of the GTS tables through Table 32 develop similarly. The rest of this section is devoted to applying the definitions and working out accompanying conditions for the three GTS.

Primarily the above GTS function notation is abbreviated as TS1, TS2 or TS3. The factor ½ in TS1 results in sequences term spacing being closer together numerically in comparison to the sequences term spacing of TS2 and TS3. The generalization of the step number is $s_j(m) = r(m, n) – y(m, n)$. $s_j(m)$ identify those steps in the Staircase algorithm that produce primitive triples; all other steps contribute redundant information. The numbering of the GTS as TS1, TS2 and TS3 was chosen so that even j would correspond to GTS tables with even m and the two odd j would correspond to GTS tables with the same odd m in the 3 member cycle of triples sequences considered at the end of Sec. 3.1 producing two table components per odd table number.

$y(m, n)$ determines the initial values of n, n_1, n_2 and n_3, since $y(m, n) = n^2 – m^2$ must be greater than 0. This results in the following three rules for n_1, n_2, and n_3:

TS1: m odd, n odd, $n^2 – m^2 > 0$ implies $n_1 > m$ where n_1 is the first odd n after m
TS2: m even, n odd, $n^2 – m^2 > 0$ implies $n_2 > m$ where n_2 is the first odd n after m
TS3: m odd, n even, $n^2 – m^2 > 0$ implies $n_3 > m$ where n_3 is the first even n after m

n_1, n_2 and n_3 scale with m. These three general conditional rules determine the first n for any given m for sequences TSj used in the evaluation of a GTS table.

The determination of $n(\nu)$ for each table is governed by these same rules. ν are the sequence of natural numbers but $n(\nu)$ are either even or odd natural numbers. Thus $n(\nu) = 2\nu + c$. In the case of Table 1.1, TS1 applies. $m = 1$ so that odd $n(\nu)$ for which odd $n(\nu) > m = 1$ implies $n(1) = 3$. $n(1) = 2 \cdot 1 + c$ implies $c = 1$ for Table 1.1; thus $n(\nu) = 2\nu + 1$. Since 2ν is always even, c will be odd for TS1 and TS2 since $n(\nu)$ must be odd and c will be even for TS3 since $n(\nu)$ must be even. Moreover a general pattern can be observed in the GTS tables: the constant c is a function of m where for TS1 $c(m) = m$, and for TS2 and TS3 $c(m) = m - 1$.

The three elements $x(m, n)$, $y(m, n)$ and $r(m, n)$ satisfy the Pythagorean theorem: from $\{2 m n, n^2 - m^2, n^2 + m^2\}$, $(2 m n)^2 + (n^2 - m^2)^2$ sums to $(n^2 + m^2)^2$ producing the identity $r^2 = r^2$. The three rules above for n_1, n_2 and n_3 cover all cases limiting the forms of TSj to the three GTS above. The GTS above are what the systematic application of the Pythagorean theorem provides.

Which of x, y and r are odd or even is now considered. For TS1, m and n are both odd so $x(m, n) = m n$ is odd. $y(m, n) = ½ (n^2 - m^2) = ½ (n - m) (n + m)$; both factors in parentheses are even so ½ times two even factors is even. The Pythagorean theorem requires r to be odd. Thus the even sum $n^2 + m^2$ must have exactly one factor of 2 to cancel the factor of ½ in $r(m, n)$. The proof begins by defining o and e: let o stand for an odd number and let e stand for an even number. Let o_1 represent n and o_2 represent m in the expression $n^2 + m^2$. $o_1 > o_2$ since $n_1 > m$. Let e_1 be the even number before o_1 and e_2 be the even number before o_2. Then $o_1 = e_1 + 1$ and $o_2 = e_2 + 1$. Squaring o_1 and o_2, forming them into the sum $o_1^2 + o_2^2$ and simplifying yields $2 (½ (e_1^2 + e_2^2) + e_1 + e_2 + 1)$, exactly twice an odd number since $½ (e_1^2 + e_2^2)$ is still even. Thus $r = ½ (n^2 + m^2)$ is odd for all TS1(s(m), n). For TS2, m is even and n odd. $x(m, n) = 2 m n$ is even. $r(m, n) = n^2 + m^2$ is odd since m is even and n is odd. Thus y is odd as required by the Pythagorean theorem, but also $y(m, n) = n^2 - m^2$ is odd since m is even and n is odd. For TS3, m is odd and n is even so again $x(m, n) = 2 m n$ is even and $y(m, n)$ and $r(m, n)$ are odd by similar reasoning used just above. Thus $r(m, n)$ is always odd for all three GTS. Thus the hypotenuses of all primitive Pythagorean triangles are odd natural numbers (> 3). Thus α is odd for non-primitive triples produced by the GTS. The cases of m and n having common factors are taken up in Sec. D.4. In the definition of the search algorithm Latchkey in Sec. E.6 it was possible to take advantage of the fact that m is even for TS2 and m is odd for TS3 allowing the interleaving TS2 and TS3 in the programming of Latchkey.

The step number is determined by the triples elements' expressions contained in TS1, TS2 and TS3. For TS1 $s_1(m) = r(m, n) - y(m, n) = ½ (n^2 + m^2) - ½ (n^2 - m^2) = m^2$ yielding a sequence of odd squares of starting at m = 1: 1, 9, 25, 49, …. For TS2 $s_2(m) = r(m, n) - y(m, n) = (n^2 + m^2) - (n^2 - m^2) = 2 m^2$ yielding the sequence of twice the square of the even m starting at m = 2: 8, 32, 72, …. For TS3 $s_3(m) = r(m, n) - y(m, n) = (n^2 + m^2) - (n^2 - m^2) = 2 m^2$ yielding the sequence of twice the square of the odd m starting at m = 1: 2, 18, 50, 98, …. These derivations validate the inclusion of the expressions for $s_j(m)$ in the definitions of the three GTS, TS1, TS2 and TS3 since these patterns are exhibited in GTS tables. For odd m GTS tables with two parts the step number for the second part is twice that for the first part.

◎(ν◎ and m are the keys to the systematic evaluation of the GTS. ◎(ν◎ generates the three sequences x(m, n), y(m, n) and r(m, n) through the evaluation of the functions n(ν◎ which are governed by the general rules for n_1, n_2 and n_3 arrived at above. ◎(ν◎ could be called the triples generating function. Incrementing m gives rise to an infinite sequence of tables starting with step number s(1) = 1. The step number identifies each GTS table or table component uniquely. m is the sequence of natural numbers and s(m) is a sequence with domain m. TSj expressions are uniquely determined by m and s(m). m is an apt table number since it is the sequence of natural numbers while the system accommodates two odd table number components with different step numbers.

As shown above in Sec. B, triples come in symmetric pairs. Given one of the pair, it is useful to be able to locate the other triple of the pair. For a given triple {x, y, r} the natural number solution point is (x, y). Thus the symmetric triple is {y, x, r}. The step number for this symmetric triple is labeled s_s where the subscript s stands for symmetric: $s_s = r – y$. The corresponding table number is labeled m_s. There are two cases. For the case that {x, y, r} is generated by GTS TS1, the step number of {y, x, r} is then $(r – x) = 2\ m_s^2$ since it is generated by either TS2 or TS3. Thus $m_s = \sqrt{((r – x)/2)}$. For the case that {x, y, r} is generated by GTS TS2 or TS3, the step number for {y, x, r} is $(r – x) = m_s^2$ since it is generated by only TS1; thus $m_s = \sqrt{(r – x)}$. Reproduced in the column heads these equations were programmed in Excel to compute column s_s and column m_s in the GTS tables enabling the search for any symmetric triple.

C.3 Description of the General Triples Sequences Tables' Contents

The 32 GTS tables are in fact 48 tables/table components. They list the first 40 triples generated by each TSj(s(m), m). As determined above, the odd numbered tables have two components: the first table component is generated by TS1 and the second table component by TS3; single component even numbered tables are generated by TS2. Table components share the same value of m. Each table/table component has a unique step number. These step numbers are prominently featured in the title of the table/table component. TSj, n(ν) and m value are also found in each table title. Composite m values are factored to aid in considerations of common factors of m and n.

Most columns in the GTS tables are calculated by *Excel*. The first, column *ν*, is the initial values of infinite sequence of the natural numbers, the domain of the triples. This sequence is also the domain of definition for each distinct function n(ν) for each distinct GTS table. This is followed by the column of evaluated values of n(ν), a necessary resource for identifying sums $n^2 + m^2$ corresponding to Fermat primes and composites. The next three columns are the evaluated values of the elements of the triples x, y and r. The expressions for calculating the values of x, y and r are stated in each column head.

The next two columns show the k value of Fermat's primes of the form 4 k + 1. Column *k* is computed from the relation 4 k + 1 = r. Developed below is that if r is an element of a primitive triple, then it is either a Fermat prime or a composite of Fermat primes only. That k values apply to the composites is developed in Sec. E. The next column calculates Δk by subtracting the (ν – 1)*th* value of k from the ν*th* value of k reported in row ν. There is a regular pattern in Δk depending on which TSj generates r.

Column γ is the angle of r relative to line y = x calculated using the built in *Excel* arctan function. The value calculated is closely related to the value in the last column, column *(y – x)*. When (y – x) approaches 0, γ approaches 0. The largest triple found in Sec. F is {803,760, 803761, 1136689}. θ is the angle of r relative to the x-axis; a rounded θ for {803,760, 803761, 1136689} is 45.000036°. In the GTS tables, the smallest γ along with the triple entries are in bold print: this is the crossover row for the transition from lower triples to upper triples.

Column s_s gives the step number of the table/table component where the symmetric triple resides and column m_s is the table number where symmetric triple occurs. The consideration of the calculation of s_s is outlined at the end of Sec. C.2 directly above.

Column α reports the *Excel* calculated GCD(x, y, r) of the triple's three components x, y and r; column β reports the *Excel* calculated GCD(m, n) for m and n. Both these values are 1 for primitive triples and are greater than 1 for non-primitive triples. For α > 1, then $\alpha = \beta^2$ as shown in Sec. D.3.

Column s_p is the step number that locates the PT-factor for those table entries where α and β are greater than 1. The subscript p stands for primitive. The formula $s_p = (r - y)/\alpha$ is inserted in the column heading. When α = 1, s_p matches the step number listed in the table title, a self-reference. The relation between k in column *k* in a GTS table for a non-primitive triple and the k for its PT-factor is $k_{PT} = (4\ k + 1 - \alpha)/(4\ \alpha)$.

Column *(y – x)* is negative for lower triples and positive for upper triples. The sign change coincides with the crossover for γ from lower triple to upper triple. If α > 1, then x, y and (y – x) have the common factor α.

C.4 Search of GTS Tables: Incongruent Pythagorean Triangles With Equal Areas

M. R. Schroeder[6] reported one pair of incongruent Pythagorean triangles that have the same area. The two upper primitive triples corresponding to these two triangles are {12, 35, 37} from Table 1.2 and {20, 21, 29} from Table 2. At the time of the publication of Ref. 6 this was the only known such pair of triangles. x · y is twice the area of such triangles. It is 420 for the pair above. On the worksheets of tables with odd table number m.1 a column calculating x · y was created for each such table through Table 31 (these columns do not appear in the GTS tables provided here). Tables m.1 cover exactly one of all symmetric primitive triple pairs minimizing the search for equal areas. Non-primitive triples' values of x · y are excluded for reasons considered in Sec. B. Using *Mathematica*'s `List` feature, lists of x · y values for each table were culled from the 16 tables into 16

Mathematica lists. These lists were combined into a *Mathematica* list of lists and then using the `Flatten` feature formed into a single list. The `Sort` feature was used to sort the flattened list. The following values of $x \cdot y$ were found in the full list in adjacent entries giving different triples with the same area. The list of equal neighboring values of $x \cdot y$ and the corresponding upper primitive triples for $x \cdot y < 1{,}000{,}000$ is

420: {12, 35, 37} from Table 1.2 and {20, 21, 29} from Table 2
5460: {28, 195, 197} from Table 13.1and {60, 91, 109} from Table 3.2
15960: {95, 168, 193} from Table 5.1 and {40, 399, 401} from Table 1.2
143220: {341, 420, 541} from Table 11.1 and {132, 1085, 1093} from Table 2
212520: {385, 552, 673} from Table 11.1 and {280, 759, 809} from Table 5.2
228228: {77, 2964, 2965} from Table 1.1 and {364, 627, 725} from Table 7.2
683760: {231, 2960, 2969} from Table 3.1 and {111, 6160, 6161} from Table 1.1

The first value of $x \cdot y$ is the one reported by Schroeder. For each pair in the above list $x_1 \cdot y_1 = x_2 \cdot y_2$. For example $12 \cdot 35 = 20 \cdot 21 = 420$. The above does not include all such pairs below 1,000,000 since odd table numbers for $m > 31$ are not included and $n < 41$. This search was not extended since the purpose here was to show that there is more than one pair of incongruent triangles with equal areas and to describe the method used to find them. The GTS tables were necessary and available in the systematic search described above. The GTS tables do contain all equal area incongruent Pythagorean triangles.

D. Patterns of Non-Primitive Triples in the GTS Tables

Casual inspection of all the GTS tables shows that a significant fraction of those tables have non-primitive triples. These triples exhibit distinct patterns throughout the GTS tables. A key indicator of non-primitive triples is column α. α is the GCD(x, y, r) and detects non-primitive triples. β = GCD(m, n) also detects non-primitive triples. Before finding the source of the non-primitive triples, it will be established which tables contain primitive triples only and then consider the occurrence of non-primitive triples in the remaining tables. The β = GCD(m, n) clue leads to the process of the factorization of non-primitive triples into the product of an odd natural number times a PT-factor.

D.1 GTS Table 1 Exhibit Only Primitive Triples

Table 1 exhibits two independent components: Table 1.1 and Table 1.2. It was already established in Sec. C.1 that Table 1.1 contains primitive triples only. However a second proof is offered here as a model for the next proof.

TS1 generates Table 1.1: TS1(1, n) = $\{n, \frac{1}{2}(n^2 - 1), \frac{1}{2}(n^2 + 1)\}$. $x(1, n) = n$, $y(1, n) = \frac{1}{2}(n^2 - 1) = \frac{1}{2}(n + 1)(n - 1)$ and $r(1, n) = \frac{1}{2}(n^2 + 1)$. By inspection x, y and r have no common factors and are relative prime. Thus all triples produced by TS1(1, n) are primitive triples. This makes Table 1.1 a reference table for PT-factors for many tables with table number > 2.

TS3 generates Table 1.2: TS3(2, n) = $\{2\,n, n^2 - 1, n^2 + 1\}$. The proof for TS3(2, n) is exactly the same as that for Table 1.1 since the Euclidian general expressions in Eq. 2 differ only by a factor of 2 from those in Eq. 1. Thus Table 1.2 is a reference table for PT-factors.

D.2 GTS Tables With Table Number m = 2^b Exhibit Only Primitive Triples

TS2 generates Tables with m = 2^b, where b is the infinite sequence of the natural numbers. The first such table is Table 2 at step 8. In general the generating GTS is

$$\begin{aligned} \mathrm{TS2}(2^{b+1}, n) &= \{2^{b+1}\,n,\; n^2 - 2^{2b},\; n^2 + 2^{2b}\} \\ &= \{2^{b+1}\,n,\; (n - 2^b)\,(n + 2^b),\; n^2 + 2^{2b}\}. \end{aligned} \tag{3}$$

By inspection x, y and r are relative primes. Thus the triples produced are primitive triples for all tables with m = 2^b in general and Tables 2, 4, 8, 16 and 32 in this paper. In general Table 1 and this infinite sequence of tables form reference tables for PT-factors. All the rest of the even m tables contain non-primitive triples. In this paper Tables 6, 10, 12, 14, 18, 20, 22, 24, 26, 28, and 30 contain non-primitive triples.

D.3 GTS Tables with Table Number a Prime p Greater Than 2

When table number m is a prime number p > 2 and as n takes on values prescribed by n(ν) for a given table, then p will unavoidably be a periodic factor of n. When this occurs, it will be shown that the three components of the Euclidian general term will share factors of p^2. Factoring p^2 from the general term results in a PT-triple with $\alpha = p^2$.

Consider a GTS table with table numbers m.1, i.e component Table m.1, where m = p where prime p > 2 and of course is odd. There are two generating GTS to consider: TS1 and TS3. m and n are both odd for TS1 and s(m) = m^2. Column β in the GTS tables is the GCD(m, n). In this case m = β = p. n is always greater than m. Thus, if there is a common factor, it must be m = p. Since n increases progressing downward in a table, m is a factor of n in sequential pattern: p, 2 p, 3 p,… with periodicity p. There will be a first n with n = p · l where l is an odd natural number since n is odd. As an example, the first value of n in Table 3.1 is 5 in row 1 so that first value of n in Table 3.1 with a factor of 3 is 9 in row 4. p is 3 and ◎ = 3 for n = 9, l = 3 for n = 15, l = 7 for n = 21 and so on with l taking the odd natural numbers starting at 3. For the general case, substituting m = p and n = p · ◎ into TS1 yields

$$TS1(m^2, m) = \{m\,n, \tfrac{1}{2}(n^2 - m^2), \tfrac{1}{2}(n^2 + m^2)\}$$

$$= \{p\,(p\,l), \tfrac{1}{2}(p^2 l^2 - p^2), \tfrac{1}{2}(p^2 l^2 + p^2)\} \qquad (4)$$

$$= p^2 \{l, \tfrac{1}{2}(l^2 - 1), \tfrac{1}{2}(l^2 + 1)\} = \alpha\, TS1(1, 1)$$

In the last step in Eq. 4 the form $\{l, \frac{1}{2}(l^2 - 1), \frac{1}{2}(l^2 + 1)\}$ is identical to step 1 GTS TS1(1, l), Eq. 1. I.e., the PT-factor is located in Table 1.1. The resultant triples in variable l have m = 1 (with multiplicative constant ½) and l being odd natural numbers starting at 3. This result is general since p can be any odd prime. The last line of Eq. 4 implies $\alpha = \beta^2 = p^2$ since the non-primitive triple is α times the PT-factor. The relation $\alpha = \beta^2 = p^2$ is general since the first line in Eq. 4 is general. With ever larger prime m = p, the separation between the non-primitive triples governed by the periodicity of p gets ever larger in agreement with prime m > 2 table numbers in the GTS tables. These results apply to the first component of Tables 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31.

Tables with table number m.2, i.e. component Table m.2, are generated by GTS TS3 with $s(p) = 2\,p^2$. m = p is odd as above but the composite n are even. Again consider the example p = 3 in Table 3.2. n takes values 2, 4, 6, 8, 10, 12,…. n is divisible by p at values 6, 12, 18, …. If $n = p \cdot l$, l takes values 2, 4, 6, …. Substituting $n = p \cdot l$ and m = p into TS3 yields

$$TS3(2\,m^2, n) = \{2\,m\,n, n^2 - m^2, n^2 - m^2\}$$

$$= p^2 \{2\,l, l^2 - 1, l^2 + 1\} = \alpha\, TS3(2, l) \qquad (5)$$

where l are even. The resultant PT- factor in l has m = 1 so it is step 2 GTS TS3(2, n): the triple in the last line in Eq. 5 is identical to Eq. 2. Thus the sequence of primitive triples in Table 1.2 are the PT-factors for TS3(s(m), n) for all table numbers m with prime m > 2. These results apply to the second component of Tables 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31. Looking at these tables consulting columns α, β and s_p shows that the non-primitive triples occur with period p for both components of each table. If an entry in a table is a primitive triple, s_p self references the table being considered; if the entry is a non-primitive triple, s_p is the step number of the PT-factor. Division of the non-primitive triple by α gives the PT-factor that will be found in the table with step number s_p. As the common factors get more complicated with the advent of composite m, s_p reveals the location of all PT-triples associated with any non-primitive triple that resides in a GTS table.

D.4 The Remaining Tables Analyzed Using Step Number s_p

The GTS tables above had m values of primes p. The remaining tables have composite m. Composite m has the following forms: 2 p, 4 p, 8 p, 2 p^2, $p_1 \cdot p_2$, 2 $p_1 \cdot p_2$ and p^3.

Consider tables with m = 2 p. The applicable tables are Tables 6 at step 72, 10 at step 200, 14 at step 392, 22 at step 968, and 26 at step 1352. In all cases the common factor is p. Table 6 is similar to Table 3, Table 10 is similar to Table 5, etc. A major similarity is that the period of non-primitive triples is p (the factor 2 does not affect the period). The factor 2 sets the position of the non-primitive entries in the tables. The tables where the PT-factors reside are found in column s_p: all 5 tables' PT-factors appear in step 8 Table 2.

Consider tables where m = 4 p. These are Tables 12, 20 and 28. Table 12 is similar to Table 3, Table 20 to Table 5 and Table 28 to Table 7. This is the same pattern as for m = 2 p. For Table 12 the period of the separation of non-primitive triples is 3; s_p places the PT-factors in step 32 Table 4. Similar patterns hold for Tables 20 and 28.

The last power of 2 applies to Table 24: $m = 8 \cdot 3 = 24$. The periodicity of the non-primitive triples in Table 24 is 3. s_p places the PT-factors in Step 128 Table 8.

Consider $m = p \cdot p$. The two tables satisfying this condition are 9 and 25. The periodicity of non-primitive triples in Table 9 are 3 and $9 = 3^2$, the first case of multiple periods. The PT-factors occur at step 9 Table 3.1 and step 1 Table 1.1 for Table 9.1 and at step 18 Table 3.2 and step 2 Table 1.2 for Table 9.2. Similar patterns hold for Table 25.

$m = 2 \cdot p^2$ occurs in Table 18. The periods of the non-primitive triples are 3 and 9. The PT-factors are found in step 8 Table 2 and step 72 Table 6.

In Table 15 there are two different prime factors so that this case is more complex. For Table 15 there are periods of 3, 5 and 15. This is the first case of 3 periods. PT-factors appear at steps 1, 9 and 25 for Table 15.1 and steps 2, 18 and 50 for Table 15.2. The number of primitive triples is reduced noticeably. For $m = 30 = 2 \cdot 15$ includes a power 2 in the composite. Table 30 exhibits the same three periods as Table 15: 3, 5 and 15. The PT-factors appear at steps 200, 72 and 8.

The last table to be considered is $m = p^3 = 27$. s_p in Table 27 shows that it has periods 3, $9 = 3^2$ and $27 = 3^3$ and with PT-factors appearing at steps 1, 9 and 81 for Table 27.1 and at steps 2, 18 and 162 for Table 27.2.

E. Two Stage Rearrangement of r(m, n) and the Search Algorithm Latchkey

r(m, n) computed by the three GTS resulting in Tables 1 – 32 constitute 48 different infinite sequences of steadily increasing values of evaluated r(m, n) where distinct values of m and n are registered in and can be retrieved from the GTS tables for each r(m, n). In this section the three general sequences of r(m, n) restricted to primitive triples only are combined and rearranged into a single sequence called the $r(\nu_r)$ sequence. This first stage of rearrangement is the union of the three GTS while sorting the union by increasing size. The sequences of evaluated r(m, n) in Tables 1 – 32 are ordered by $n(\nu)$ in each GTS table. The restriction to primitive triples is a consequence of the considerations at the end of Sec. B. This process creates a new and different sequence term rule for this new sequence: the next term is the next biggest r(m, n) from one of the other 47 steps. As will be shown from the developments in Secs. E.1, E.2 and E.3, r(m, n) are either Fermat primes or composites of Fermat primes only. The first stage reordering in Sec. E.2 ends with the separation of the sequence $r(\nu_r)$ into the sequence of Fermat primes with their own domain of definition ν_p and then the sequence of the composites of Fermat primes with their own domain of definition ν_c. The general terms for these three sequences is next largest r(m, n), the next largest Fermat prime or the next largest composite of Fermat primes.

The second stage is the reordering of the $r(\nu_r)$ sequence. This is accomplished by forming powers of Fermat primes and the Fermat composites each and all into systematic term sequences. These new systematic term sequences are found from the information developed in Sec. E.2 including Table RPC1, Sec. E.3 including Table RPC2 and fully developed in Sec. E.5 including Table FSTS. The general term of each sequence is the product of one or more distinct primes, some of which may be powers of natural numbers, times a prime distinct from the others, possibly of a power, that serves as the general term variable. Details on general terms follow in Sec. E.5.

In Sec. E.6 search algorithm Latchkey is defined. Latchkey computes all primitive triples that correspond to a given value of r(m, n) by finding the values of m, n, x(m, n) and y(m, n) from the values of r(m, n) by employing the Pythagorean Theorem, the step number expression from Staircase and general triples expressions for x, y and r.

E.1 Fermat Primes of the Form 4 k + 1 and Direchlet Primes of the Form 4 k – 1

The following list is a quartet of odd number sequence generators:

$$\omega_{11}(k) = 2k - 1 \qquad \omega_{21}(k) = 4k - 1$$
$$\omega_{12}(k) = 2k + 1 \qquad \omega_{22}(k) = 4k + 1$$

The functions $\omega_{ij}(k)$ have the natural numbers k for their domains. $\omega_{11}(k)$ generates the sequence of odd numbers from 1 to ∞. $\omega_{12}(k)$ generates the sequence of odd numbers from 3 to ∞. $\omega_{21}(◎)$ generates the sequence of every other odd number from 3 to ∞. $\omega_{22}(k)$ generates the sequence of every other odd number from 5 to ∞. The union of ω_{21} and ω_{22} is ω_{12}.

Fermat discovered what are called Fermat primes of the form 4 k + 1 which are imbedded in the odd numbers generated by $\omega_{22}(k)$ and are formed by the sum of the squares of two distinct natural numbers. As shown in Sec. E.3, GTS TS2 and TS3 generate these sums. The following *Mathematica* code produces Fermat primes, $p(\nu_p)$, from $\omega_{22}(k) <$ 3000 in the form of a *Mathematica* list.

```
fermatPrimeList = Select[Table[4 k + 1, {k, 1, 749}], PrimeQ]
     {5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97, 101, 109, 113, 137, 149, 157, 173, 181, 193, 197,
      229, 233, 241, 257, 269, 277, 281, 293, 313, 317, 337, 349, 353, 373, 389, 397, 401, 409,
      421, 433, 449, 457, 461, 509, 521, 541, 557, 569, 577, 593, 601, 613, 617, 641, 653, 661,
      673, 677, 701, 709, 733, 757, 761, 769, 773, 797, 809, 821, 829, 853, 857, 877, 881, 929,
      937, 941, 953, 977, 997, 1009, 1013, 1021, 1033, 1049, 1061, 1069, 1093, 1097, 1109, 1117,
      1129, 1153, 1181, 1193, 1201, 1213, 1217, 1229, 1237, 1249, 1277, 1289, 1297, 1301, 1321,
      1361, 1373, 1381, 1409, 1429, 1433, 1453, 1481, 1489, 1493, 1549, 1553, 1597, 1601, 1609,
      1613, 1621, 1637, 1657, 1669, 1693, 1697, 1709, 1721, 1733, 1741, 1753, 1777, 1789, 1801,
      1861, 1873, 1877, 1889, 1901, 1913, 1933, 1949, 1973, 1993, 1997, 2017, 2029, 2053, 2069,
      2081, 2089, 2113, 2129, 2137, 2141, 2153, 2161, 2213, 2221, 2237, 2269, 2273, 2281, 2293,
      2297, 2309, 2333, 2341, 2357, 2377, 2381, 2389, 2393, 2417, 2437, 2441, 2473, 2477, 2521,
      2549, 2557, 2593, 2609, 2617, 2621, 2633, 2657, 2677, 2689, 2693, 2713, 2729, 2741, 2749,
      2753, 2777, 2789, 2797, 2801, 2833, 2837, 2857, 2861, 2897, 2909, 2917, 2953, 2957, 2969}
```

Dirichlet proved that primes of the form 4 k – 1 in $\omega_{21}(\circledcirc)$ are infinite and that primes of the form 4 k + 1 from $\omega_{22}(\circledcirc)$ (i.e. Fermat's primes of the form 4 k + 1) are infinite.[7] The primes of ω_{21} are called Dirichlet primes of the form 4 k – 1.[7] They are 3, 7, 11, 19, 23, 31, …. Dirichlet primes appear in r(m, n) only when m and n have Dirichlet primes as common factors (see Sec. D), i.e. when table numbers are Dirichlet primes or when Dirichlet prime(s) is/are factor(s) of table numbers m. Consequently the corresponding triples are non-primitive. As shown in Sec. D the common factors appear as α and β greater than 1 only whereas $\alpha = \beta = 1$ for the corresponding PT-factors. For example in Table 7.1 at row 28 the non-primitive triple {441, 1960, 2009} occurs where s_p locates the PT-factor as primitive triple {9, 40, 41} in Table 1.1. r(m, n) is never solely a Dirichlet prime. Thus Dirichlet primes have no natural number solution points interior to the coordinate axes on a circle of radius equal to a composite with one or more Dirichlet primes in the Cartesian plane. Fermat primes and composites of Fermat primes can be represented as polar vectors in the Cartesian plane terminating on the circle at one of the natural number solution points as shown in Fig. 1. They never lie on the coordinate axes. Since Dirichlet primes do not correspond to triples, they are represented as polar vectors that coincide with the coordinate axes. Twin primes have one prime a Dirichlet prime and the other prime a Fermat prime, their difference being 2. The infinite $r(\nu_r)$ sequence is the union the infinite sequence of all primes in the infinite sequence ω_{22} along with the infinite sequence of all composites of Fermat primes, both partialy listed in Table RPC1 below.

Values of k for r(m, n) are placed next to column *r* in the GTS tables. These k values are unique for r in TS2 or TS3 evaluations and also separately unique for the same r in TS1 evaluations.

E.2 First Stage Reordering and Separation of r(m, n) in the GTS Into Three Sequences

In what follows r(m, n) from the three GTS are formed into a single infinite sequence. This development of necessity requires a second notation for the r values in the GTS tables suitable for a new single sequence of the terms of the three GTS r(m, n) sequences. This new sequence has a single domain of definition ν_r, the sequence of natural numbers, that enumerates the new terms of the new sequence consecutively. The new function notation for r values is $r(\nu_r)$. Each value of $r(\nu_r)$ will have a unique value of the r

= r(m, n) in the GTS tables where nothing is lost, gained nor duplicated. The r values in Table RPC1 are $r(\nu_r)$. The r values in Table RPC2 are both $r(\nu_r)$ and r(m, n). The correspondences in Table RPC2 are clearly delineated and can be verified by cross-checking between the GTS and Table RPC1.

The sorted union of r(m, n) from all GTS is formed by the following procedure: r(m, n) in the GTS, restricted to primitive triples only, are systematically placed in a sequence of ascending magnitude labeled $r(\nu_r)$ in Table RPC1 forming the $r(\nu_r)$ sequence. The first value of r of the $r(\nu_r)$ sequence is $r = r(\nu_r) = r(m, n)$ where $\nu_r = 1$, m = 1, n = 3 so that r = r(1) = r(1, 3) = 5 in the first row of Table 1.1. Successive entries are the next larger r until the largest r < 3000. This systematic selection is the general rule for the infinite sequence $r(\nu_r)$. The domain of definition is the natural numbers in column ν_r. To limit Table RPC1 to a manageable size, only upper primitive triples from the values of r(m, n) are entered into Table RPC1. The corresponding lower primitive triples can be located using s_s in the GTS tables. The value 3000 is arbitrary but was deemed adequate to reveal the several new patterns that become apparent below while keeping Table RPC1 relatively small. $r(\nu_r)$ are only a very small fraction of all evaluated r(m, n) in the 32 GTS tables with the upper bound on r of $\nu = 40$ in those tables.

Some members of the $r(\nu_r)$ sequence are prime numbers and the rest are composites. The primes in $r(\nu_r)$ are Fermat primes of the form 4 k + 1 (see Sec. E.1). The rest of $r(\nu_r)$ are composites of Fermat primes only. These composites are named Fermat composites since their factors are entirely composed of Fermat primes found in $\omega_{22}(k)$. The existence of Fermat composites came as a surprise likely since Table RPC1 was assembled before the infinite sequence $\omega_{22}(k)$ was created. So Fermat composites share a property with Fermat primes in addition to being composed of Fermat primes: they are part of the form 4 k + 1 found in the infinite sequence $\omega_{22}(k)$. As examples, the first square of a Fermat prime is 5 squared, i.e. 25, the first Fermat composite, where $k = (25 - 1)/4 = 6$. The first composite with distinct Fermat primes is $65 = 5 \cdot 13$ where $k = (65 - 1)/4 = 16$. Both 25 and 65 are found in Table RPC1. What is new is that both 25 and 65 are the hypotenuse of a primitive triples' Pythagorean triangle. From here on the phrase *of the form 4 k + 1* is implied for the names Fermat primes and Fermat composites.

The sequence $r(\nu_r)$ is found in the column *r(* ν_r *)* in Table RPC1. $r(\nu_r)$ is a single valued function resulting from the combining of the r(m, n) of TS1, TS2 and TS3 into a single sequence where each TSj has its own domain of definition ν as developed in Sec. B and Sec. C. 2. The expansion of TS1, TS2 and TS3 was executed using the distinct functions $n_j(\nu)$ associated with each GTS TSj. Besides counting the entries and forming the domain of definition of $r(\nu_r)$, ν_r is a row number throughout Table RPC1 hiding in plain sight. Table RPC1 has two sets of columns where the first set on the left is the start of the $r(\nu_r)$ listings and where the listings then continue on the right side of Table RPC1 with column heads duplicated; this arrangement follows throughout Table RPC1.

The notation for the Fermat primes is $p(\nu_p)$ where ν_p is the domain of definition for the sequence $p(\nu_p)$. ◎◎ν_p is found in column ν_p. Values of $p(\nu_p)$ are found only in column *r(*ν_r*)* in Table RCP1. The notation for the Fermat composites is c(ν_c) found in the column *c(*ν_c*)* in the form of factored Fermat composites. ν_c is found in the column ν_c. Table RPC1, where p(ν_p) and c(ν_c) are separated out from the r(ν_r) sequence, results in that table having three different infinite sequences.

The values of both p(ν_p) and c(ν_c) (where c(ν_c) is not factored) are found in the column *r(*ν_s*)*. The sequence of Fermat primes in the r(ν_r) is complete to the value 3000 and the list of all possible Fermat composites to the value 3000 is also complete. There are no missing Fermat primes and no missing Fermat composites nor are there any entries from outside these two categories in column *r(*ν_s*)* in Table RPC1, all from the GTS tables. A further elaboration of r(ν_r) is found in Sec. E.3.

Table RPC1 registers only values of r for upper primitive triples from the GTS tables in column *r(*ν_r*)*. Values from both upper and lower primitive triples for a given value of r are included in Table RPC2 as described directly below. Values of ν_p in column ν_p and ν_c in column ν_c in Table RPC1 are mutually exclusive since r(ν_r) is either a Fermat prime or a Fermat composite. So column ν_p entries signal those values of r(ν_r) that are Fermat primes and column ν_c entries signal those values of r(ν_r) that are Fermat composites. Column *Upper Table* in Table RPC1 shows GTS table numbers corresponding to upper primitive triples values of r(ν_r) appear in Table RPC1. Those tables are connected to tables with lower triples corresponding to the upper triples through column s_s in the GTS tables. Most r(ν_r) appear in one GTS table, some in two GTS tables and a few in four GTS tables. Extension to more than three distinct Fermat prime factors in a Fermat composite are found in Sec. E.3, Sec. E.5, Sec. E.6 and Sec. E. 7.

Table RPC1 reveals two new patterns: the list of Fermat primes of the form 4 k + 1 and the list of all composites of those primes. There are further patterns in the composites sequence to be elaborated in Sec. E.5. The initial listings of the r(ν_r) in Table RPC1 are fully parsed in Sec. E.3. The initial listings of non-primitive triples values of r(m, n) in the GTS are parsed in Sec. E.4.

E.3 Parsing r(m, n) In Table RPC2: Generators of Fermat Primes and Fermat Composites

Table RPC2 is a detailed parsing of r(m, n) for the first 20 entries in Table RPC1. Both upper and lower primitive triples are listed in Table RPC2, these symmetric triples taken from the GTS tables using m_s. The columns in Table RPC2 are row number, sequence term variables ν_r, ν_p, and ν_c, r, GTS table numbers, the distinct primitive triple corresponding to each particular r(m, n) = r(ν_r), the upper/lower label for triples in column U/L, the Fermat prime factors of Fermat composites where applicable, the values of m and n for each particular r(m, n) taken from the GTS tables, unevaluated r(m, n) with the values of m and n substituted into the applicable expression for each particular r(m, n) (i.e. ($n^2 + m^2$) or ½ ($n^2 + m^2$)), and lastly the GTS label that applies, i.e., TS1, TS2 or TS3. Half the entries in Table RPC2 correspond to TS1, 20 in all. 9 entries correspond to TS2 and 11 entries correspond to TS3. The total for TS2 and TS3 is 20, matching the 20 corresponding to TS1. This represents the equal balance of TS1 and the combined action of TS2 and TS3 alluded to in the brief description of search algorithm Latchkey above and is further considered in Sec. E.6.

For each prime value of r there are two unevaluated r(m, n) that generate r for primitive triple pairs listed in column *r(m, n)* in Table RPC2. One of the two versions is r(m, n) = ($n^2 + m^2$) where m is odd and n even natural numbers or *visa versa*. These primes are the Fermat primes of the form 4 ◎ + 1 originally discovered by Fermat as noted above. r(m, n) from GTS TS2 and TS3 generate these primes. Thus TS2 and TS3 populate half the Fermat primes in column *r* of the GTS tables, and in Table RCP1 and Table RPC2. But there is a second version of r(m, n): r(m, n) = ½ ($n^2 + m^2$) from TS1 where m and n are both odd natural numbers. As shown in Sec. C.2 for this case, $n^2 + m^2$ is even since both m and n are odd. It was proved there that $n^2 + m^2$ for TS1 has exactly one factor of 2 which is cancelled by the factor ½ so that ½ ($n^2 + m^2$) is odd. TS1 generates the same complete set of Fermat primes of the form 4 ◎ + 1 as those produced by TS2 and TS3 together. Table RPC2 shows that there are two ways to form the same Fermat prime. This condition satisfies the requirement that two different expressions that form Fermat primes are necessary to produce Pip pairs as considered in Sec. B above. There must be two ways to obtain 4 k + 1 for these cases provided by the three GTS.

The same considerations apply to the powers of Fermat primes and Fermat composites. Entries in Table RPC1 and Table RPC2 that are Fermat powers or Fermat composites are produced by r(m, n) from one of the three GTS. The first case of a power of a Fermat prime is 25, the square of Fermat prime 5. Table RPC2 shows that the primitive triple pair corresponding to 25 is {7, 24, 25} from TS1 and {24, 7, 25} from TS3. From Table 1.1, m = 1 and n = 7 so that r = r(1, 7) = ½($7^2 + 1^2$) = 25 = 5^2. For the lower primitive triple {24, 7, 25} in Table 3.2, m = 3 and n = 4 so that r(3, 4) = $4^2 + 3^2$ = 25. 25 is the only prime square in Table RPC2 but consecutive Fermat prime squares through 53^2 for r < 3000 are found in Table RPC1 and also in Table FSTS considered below.

The first Fermat composite is the product of two distinct Fermat primes: p(◎) · p(◎) where ◎ and ◎ are natural numbers with ◎ < ◎. There are two in Table RPC2, 65 = 5 · 13 and 85 = 5 · 17. The case of r = 65 is treated here. There are four distinct r(m, n) in Table RPC2 for r = 65 that have two distinct primitive triple pairs. The upper primitive triples for 65 are {16, 63, 65} and {33, 56, 65}. The list of the four triples corresponding to r = 65 = 4 k + 1 where k = 16 with information culled from the GTS tables is:

Triple	Table	m	n	k	r(m, n)	$x^2 + y^2 = r^2$
{16, 63, 65}	1.2	1	8	16	r(1, 8) = ($8^2 + 1^2$) = 65	$16^2 + 63^2 = 65^2$
{63, 16, 65}	7.1	7	9	16	r(7, 9) = ½($9^2 + 7^2$) = 65	$63^2 + 16^2 = 65^2$
{33, 56, 65}	3.1	3	11	16	r(3, 11) = ½($11^2 + 3^2$) = 65	$33^2 + 56^2 = 65^2$
{56, 33, 65}	4	4	7	16	r(4, 7) = ($7^2 + 4^2$) = 65	$56^2 + 33^2 = 65^2$

There are four pairs of values of m and n that form the same r that are consistent with the same k from the relation between k and r: $k(m, n) = (r(m, n) - 1)/4$. Fig. 3 shows the two pairs of Pythagorean triangles for composite r = 65. The first pair is OX_2U_1 and OX_3L_1 and the second pair are OX_1U_2 and OX_4L_2. Thus the four polar vectors that locate the four natural number solution points relative to the origin on a circle of radius equal to a Fermat composite play the same role as they do for Fermat primes: they point to natural number solution points on the curve $x^2 + y^2 = r^2$ when r is a Fermat composite with two distinct Fermat primes. These four vectors correspond to the hypotenuses of the triangles in Fig. 3. The Pythagorean theorem holds that for a Pythagorean triangle with hypotenuse of length equal to a Fermat prime, the sum of the squares of the orthogonal sides equals the square of the hypotenuse. But Fermat composites with two different factors of Fermat primes correspond to two different Pythagorean triangles with the same hypotenuse results in the square of the hypotenuse being equal to the sum of two different squares: $65^2 = 16^2 + 63^2 = 33^2 + 56^2$. This was the second surprise. Three distinct Fermat primes in a composite produces four sums of squares for the hypotenuse and so on. The search algorithm Latchkey was designed to find the primitive triples of any composite of Fermat primes.

Fermat and his correspondents considered natural number composites that were identical to two different sums of the squares of natural numbers. Fermat and then Euler developed methods for factoring such numbers.[8] An example considered by them is $221 = 10^2 + 11^2 = 5^2 + 14^2$. In Table RPC1 $r(30) = c(9) = 221 = 13 \cdot 17$. Although they all knew about Fermat primes of the form 4 k + 1, they did not recognize that the Fermat composites of the form 4 k + 1 were similar kinds of numbers. They did not imagine that either kind, i.e. primes or composites, are the hypotenuses of primitive Pythagorean triangles. [8]

Table RPC1 has five cases of Fermat composites with three distinct primitive triples produced by r(m, n). Those cases are $1105 = 5 \cdot 13 \cdot 17$, $1885 = 5 \cdot 13 \cdot 29$, $2405 = 5 \cdot 13 \cdot 37$, $2465 = 5 \cdot 17 \cdot 29$, and $2665 = 5 \cdot 13 \cdot 41$. Table RPC1 shows four table numbers for the upper triples, so using column m_s in the GTS tables provides upper and lower triples in eight tables, twice the number for r(m, n) = 65: see Table LS1 for all triples of 1105. Fig. 4 exhibits the first quadrant graph of the circle of radius 2665. There are four sums of two squares equal to 2665 and four one-half times the sums of two squares equal to 2665. There are four upper Pips on that circle. Using the coordinates of the four Pips representing these triples, then $73^2 + 2664^2 = 816^2 + 2537^2 = 1207^2 + 2376^2 = 1824^2 + 1943^2 = 2665^2$. There are four upper incongruent Pythagorean triangles with hypotenuse r = 2665.

So primitive Pythagorean triangles have hypotenuses of length Fermat primes or composites of Fermat primes. Dirichlet primes of the form 4 k – 1 are not found as lengths of the hypotenuse of right triangles with natural number sides. The square of Dirichet primes are not the sum of the squares of two other natural numbers.

More examples of four or more distinct prime factors of Fermat composites can be found in Sec. E.5, Sec E.6 and Sec. E.7.

E.4 Table RPC3: Parsing r(m, n) From The GTS Tables For Non-Primitive Triples

Table RPC3 contains the first 40 non-primitive triples ordered by size taken from GTS Tables 1 – 32. The structure of this table is similar to the structure of Table RPC2. Unevaluated r(m, n) reveal the details of how non-primitive triples emerge from r(m, n): m and n have common factors as analyzed in Sec. D. The common factors are recognizable by inspecting columns containing m and n. α calculated in the GTS tables was factored out of the unevaluated r(m, n) in Table RPC3 in column *r(m, n) -> $\alpha \cdot$ PT-factor* (see also Sec. D). The last column labeled r_{PT} records the r value for the PT-factor entries in column *r(m, n) -> $\alpha \cdot$ PT-factor*. The non-primitive triples in Table RPC3 have factors that are either Dirichlet primes of the form 4 k – 1 and/or Fermat primes of the form 4 k + 1: Dirichlet primes are 3, 7 and 11 and the Fermat prime is 5. Table 15 has prime common factors 3 and 5. Table RPC3 has entry r = 585 which is similar to r = 65 and r = 85 in Table RPC2. There are four entries for r = 585 with $\beta = 3$ and four PT-factor triples for r = 65 in Table RPC2.

E.5 Second Stage Reordering: Fermat Systematic Term Sequences

Table RPC1 was formed by first ordering a sequence of r(m, n) in the GTS in TS1, TS2 and TS3 by increasing size, and then by separating out the Fermat primes, $p(\nu_p)$, as a second sequence and then forming the Fermat composites, $c(\nu_c)$, as a third sequence from the elements of $r(\nu_r)$. The general term for all three sequences is the next larger value of the sequence where ν_r, ν_p and ν_c provide the natural number domains for $r(\nu_r)$, $p(\nu_p)$, and $c(\nu_c)$ as required for sequences.

Table FSTS is a rearrangement of Table RPC1 where $r(\nu_r)$ is not included and where all sequences are expressed entirely in terms of $p(\nu_p)$, i.e. Fermat primes. The second column in Table FSTS is the general term of each Fermat systematic term sequence (FSTS). Each general term is an expression involving combinations of $p(\nu_p)$. The first row is the Fermat primes sequence. All further rows have general terms of either powers of Fermat primes or composites of Fermat primes starting with distinct pairs of Fermat primes and so on. The arrangements in the rows of Table FSTS were suggested by the patterns noticed in Table RPC1. These distinct patterns in the values of $c(\nu_c)$ in Table RPC1 are entered in the second column of Table FSTS along with the primes and the powers of primes.

The first powers of Fermat primes in the Table FSTS occur in row 2 as the squares of the Fermat primes. Row 2 sequence has squares 25, 169, 841, … terminating at 2809 < 3000 since Table RPC1 is restricted to r < 3000. Row 3 patterns are the first Fermat prime times each larger Fermat prime taken in order. The truncated sequence is 65, 85, 145, 185, 205,… ending at 2965. So row 2 general term systematically creates the sequence of squares of the Fermat primes. Rows 3 through 8 general terms systematically create the distinct pairs of Fermat primes and so on. Table FSTS shows how Fermat systematic terms sequences (FSTS) are formed. The FSTS then are ordinary sequences with well defined general terms. The FSTS general terms are combinations of Fermat primes taken one at a time, two at a time, three at a time, four at a time and so on. The different distinct Fermat primes may have any power 0, 1, 2, 3, … as required by the pattern at hand. What drives this second stage reordering is that there are easily discernable patterns of Fermat composites that can be found in Table RPC1. These patterns are the basis of an infinite number of infinite sequences that are a new and different way of finding and organizing the Fermat composites independent of the GTS generators TS1, TS2 and TS3 and the sequence $r(\nu_r)$ in Table RPC1. All possible patterns that Fermat composites can take are considered within the scope of a greatly expanded version of Table FSTS. The rules restricting ν_p from taking values already taken in the formation of a Fermat composite general term are incorporated in the last factor, the general term variable. The natural number powers of single distinct Fermat primes are of course exempted from such a restriction since their general terms involve only one Fermat prime at a time. How the general Fermat composite (GFC) fills out the systematic terms sequences listed in Table FSTS is demonstrated directly below.

In Table FSTS a representation of the partial development of each successive sequence is found in column *Span of* $r(\nu_r) < 3000$. That column begins with the initial term of the sequence followed by an arrow pointing to the last term for the truncated sequence restricted by the condition that r < 3000. The last term in the sequence of Fermat primes found in Table RPC1 is p(211) = 2969 < 3000. The arrow symbol is used throughout the table eliminating space that would be taken by including all of the terms of the sequence not listed there, this step being taken to keep Table FSTS to a manageable size. The missing terms from any partial sequence can be found by employing the general term as expected. This can be checked in Table RPC1 by looking for those terms in columns $r(\nu_r)$, $p(\nu_p)$, or $c(\nu_c)$.

Column *Term Count* gives the total number of entries of primes or composites for a particular row. There are 211 primes out of a total of 344 entries in Table FSTS. What is left after the primes are 133 total of powers or composites where powers or composites are less than 3000. These counts are exactly the same as those in Table RPC1.

As noted above the upper bound of 3000 for Fermat primes, Fermat primes powers and Fermat composites is arbitrary. Shrinking the upper bound would reduce the contents of Table FSTS. Enlarging the upper bound would have the opposite effect including the lengthening of the point where all partial systematic terms sequences in Table FSTS end and also systematically adding new FSTS beyond those found in Table FSTS. For example only the first terms of new sequences appear in rows 11, 14 and 15 in Table FSTS.

The next procedure is to turn the partial Fermat systematic terms sequences into infinite sequences by increasing the upper bound from 3000 to infinity. This will be facilitated by the definition of the general Fermat composite, GFC:

$$\text{GFC} = 5^{\alpha} \cdot 13^{\beta} \cdot 17^{\gamma} \cdot 29^{\delta} \cdot 37^{\varepsilon} \cdot 41^{\phi} \cdot 53^{\eta} \cdot 61^{\iota} \cdot 73^{\kappa} \cdot 89^{\lambda} \cdot 97^{\mu} \cdot \ldots \tag{6}$$

It can be used to build any Fermat composite sequence in the FSTS. This is the inverse process of factoring a Fermat composite. It is trivial to form a composite but can be effectively impossible to reduce a large enough composite to its factors. The GFC is the key to extending Table FSTS to an infinite number of infinite sequences. Any Fermat prime or any Fermat composite can be generated by the choice of natural number values of α, β, γ, δ, … in GFC. The sequence of Fermat primes can be composed by systematically proceeding through the infinite set $\{\alpha, \beta, \gamma, \delta, \ldots\}$ setting first $\alpha = 1$ and the rest of the exponents 0, then $\beta = 1$ and the rest of the exponents 0, then $\gamma = 1$ and the rest of the exponents 0 and so on, producing the sequence of Fermat primes. The systematic Fermat composites of the form $p(\alpha)\ p(\nu_P + \alpha)$ can be used to form the infinite sequence $p(\alpha)\ p(\beta)$, $p(\alpha)\ p(\gamma)$, $p(\alpha)\ p(\delta)$, …. These methods can be extended to form all 16 of the sequences in Table FSTS extending these sequences to infinity without reference to TS1, TS2 or TS3. The infinite Fermat systematic term sequences in Table FSTS can be generated independently from the GTS using Eq. 6. To complete this second stage of reordering of r(m, n), the upper bound of r and the upper bound of m are allowed to increase without bound. Table FSTS exhibits only a small fraction of all Fermat composites. Primitive triples formed from Fermat composites of the form 4 k + 1 vastly outnumber the primitive triples formed by Fermat primes of the form 4 k + 1 (as with all primes and their composites.

E.6 Search Algorithm Latchkey

The considerations above show that it would advantageous to have an algorithm that could find the primitive triples corresponding to any given r where r is a Fermat prime or a Fermat composite. Defined here is Latchkey that performs that task. Like Staircase, Latchkey is a search algorithm that uses trial calculations to find natural number solutions to the Pythagorean theorem. The structure of Latchkey is summarized in the column heads of Tables LS1, LS2 and LS3. Everything in the structure of the algorithm is a trial number except r. Although any prime or natural number composite is an eligible entry, only Fermat primes or Fermat composites produce triples in Latchkey.

Each Latchkey search table has two parallel lists of columns. On the left side of Table LA1 are alternating, intermeshed GTS TS2 and TS3, each occupying every other row. On the right side of Table LA1 is GTS TS1. GTS TS2 and TS3 have the exact same expressions for x, y and r with the same variables m and n. m is even and n odd for TS2 and m is odd and n even for TS3. As a result, this feature makes it feasible to intermesh the two GTS on the left. The structures on both sides of Table LS1 are columns for trial m. On the left side all natural number values of m are listed in column *m* starting at 1 for alternating TS2 and TS3 while on the right side TS1 requires odd m starting at 1. Refer back to Sec. C.2 for these assignments. The second column on both sides calculates the trial step number which is $s(m) = 2m^2$ on the left, the same for both TS2 and TS3, and $s(m) = m^2$ for TS1 on the right. The third column calculates trial values of y, this being an application of the important feature of Staircase, the step number: $s(m) = r(m, n) - y(m, n)$. The forth column calculates trial values of x which are the square root of $x^2 = r^2 - y^2$ from the Pythagorean theorem. This procedure is similar to Staircase: r enters the Latchkey in both columns 3 and 4. The fifth column for each side calculates trial values of $n = x/(2m)$ for TS2 and TS3 and $n = x/m$ for TS1. The column heads are labeled by the equations which calculate s, y, x and n. Both sides of Latchkey are halted while being filled down in *Excel* when their respective value of y turns negative ($y > 0$ is mandatory as per Sec. C.2). The sought after triples are the only table entries with rows showing only natural numbers. Tables LS1 through LS3 demonstrate how Latchkey provides its results. Search algorithm Latchkey is similar to search algorithm Staircase: all primitive Pythagorean triples are systematically produced.

Table LS1 shows the search for the triples that correspond to the Fermat composite 1105 listed in row 135 in Table RPC1. Table RPC1 shows that 1105 corresponds to four different symmetric pairs of primitive triples. Table LS1 contains the four pair of symmetric primitive triples for $r(135) = c(46) = 1105$. In the lower right corner of the table GCD(x, y, r) shows that all four distinct triples are primitive. r(m, n) for each of the primitive triples is produced by a unique sum of squares or one-half a unique sum of squares. Fig. 4 for r = 2665 has 4 pairs of Pips: the case for r = 1105 is similar.

The Fermat composites of the powers of Fermat prime 5 each correspond to one pair of primitive triples. 5^1 corresponds to upper primitive triple {3, 4, 5}, 5^2 to {7, 24, 25}, 5^3 to {44, 117, 125}, 5^4 to {336, 527, 625} and 5^5 to {237, 3116, 3125}. Table LS2 is Latchkey search for 5^5 where two non-primitive triples were also found. Powers of exactly one Fermat prime correspond to one symmetric pair of primitive triples.

The Fermat composite $5 \cdot 13 = 65$ is considered in Table RPC2. There are two pairs of symmetric primitive triples that correspond to hypotenuse 65 as demonstrated in Sec. E.3. Table LS3 is the application of the Latchkey algorithm to $5^3 \cdot 13 = 1625$, the first term in row 16 sequence in Table FSTS. There are two primitive triple pairs as shown Table LS3 as well as two non-primitive pairs of triples.

Composites	Upper primitive triples	Non-PT upper triples
$5 \cdot 13$	{16, 63, 65}, {33, 56, 65}	0
$5^2 \cdot 13$	{36, 323, 325}, {204, 253, 325}	1
$5 \cdot 13^2$	{116, 837, 845}, {123, 836, 845]	1
$5^3 \cdot 13$	{57, 1624, 1625}, {1113, 1184, 1625}	2
$5^2 \cdot 13^2$	{2016, 3713, 4225}, {2047, 3696, 4225}	2
$5 \cdot 13^3$	{2793, 10624, 10985}, {5656, 9417, 10985}	2
$5^4 \cdot 13^2$	{22393, 103224, 105625}, {23526, 103033, 105625}	5
$5^3 \cdot 13^3$	{7336, 274527, 274625}, {186416, 201663, 274625}	6

Several examples of $5^{\alpha} \cdot 13^{\beta}$ with their corresponding upper primitive triples are entered in the above list. $5 \cdot 13$ was taken from Sec. E.3. $5 \cdot 13^3$ is from Table LS3. The remaining examples were taken from Latchkey searches not included ($5^3 \cdot 13$ appears in Ref. 6, p. 107). Latchkey search for the last example extended to row 371. All examples correspond to two symmetric primitive pairs. The number of non-primitive upper triples varies according to the powers of 5 and 13 in a regular pattern.

Table LS4 collects the output of the Latchkey search for r = 1,185,665 = $5 \cdot 13 \cdot 17 \cdot 29 \cdot 37$ (where the exponents in Eq. 6 after factor 37 are 0). For 1,185,665, k = (1185665 – 1)/4 = 296,416. As shown in column β, computing all GCD(m, n) shows all GTS triples pairs' values m and n have common factors of 1. The Latchkey algorithm found no non-primitive triples since none of the primes in the composite had powers greater than 1. There are 16 distinct pairs of Pips and thus 16 distinct Pythagorean triangles with hypotenuse 1,185,665. There are 32 distinct evaluations of r(m, n) corresponding to the 32 pairs of m and n in Table LS4. The column $n^2 + m^2$ produces 1185665 when TS2 or TS3 applies and twice 1185665 when TS1 applies; as proved in Sec. C.2, TS1's sum $n^2 + m^2$ has exactly one factor of 2 whereas TS2's and TS3's sum $n^2 + m^2$ is always odd.

The power law count N(F) of primitive triples in evaluations of GFC by Latchkey is that the number of primitive symmetric triple pairs for Fermat composites with F distinct Fermat primes, each with any natural number exponent α, β, γ, …. is 2^F primitive triples corresponding to such composites:

$$N(F) = 2^F \tag{7}$$

Thus Latchkey applied to a given Fermat composite reveals the number of Fermat primes in the composite. The Fermat composite 1,185,665 has F = 5 so that $N(5) = 2^5 = 32$ as found in Table LS4. Knowing the number of Fermat prime factors in a Fermat composite will assist in the factoring of a Fermat composite. Fermat composites should be used with care in applications in the cryptography. Dirichlet composites are not detected by Latchkey.

A Latchkey search for triples was conducted for the last prime before 1,000,000: 999,983. Substituting r = 999,983 into Latchkey finds that there are no triples corresponding to 999,983. Thus 999,983 is not a Fermat prime but is a Dirichlet prime of the form 4 k – 1. As noted above, Dirichlet primes have no natural number solution points on the locus of the circle with a radius equal to the Dirichlet prime excluding the coordinate axes intersection points. An alternative way to establish this is to let 4 k – 1 = 999,983. Solving for k results in k = 249,996 confirming that 999,983 is a Dirichlet prime.

Consider a Fermat composite formed by two distinct Fermat primes. An example is $56{,}153 = 233 \cdot 241$. A Latchkey search of 56,153 finds two pairs of primitive triples with upper triples {3013, 56072, 56153} and {30872, 46905, 56153}. This Latchkey search found that 56153 is a composite of two distinct Fermat primes via an arithmetic calculation. The calculation does not reveal the factors but they are easier to find.

E.7 Figure 4 Exhibits Fermat Primes and Fermat Composites Produced by r(m, n)

Fig. 4 graphs the Pips of four Fermat composites and Pips of one Fermat prime. The values of x, y and r are labeled on the upper left portion above line y = x of Fig. 4 allowing reconstruction of the primitive triple pairs. x and y are contained in the upper Pip coordinate points. The values of r are labeled near the y-axis intercepts of the five circle sectors. The values of r were chosen to comfortably fit on the graph and to illustrate the primes and composites in rows 1, 2, 8, 10 and 13 in Table STS.

The first value of r, 2225, is an example from row 10 in Table FSTS. The composite $r_1 = 2225 = r(259) = c(98) = 5^2 \cdot 89$. The Latchkey algorithm calculates the two primitive triple pairs located on circle $r_1 = 2225$: the two upper primitive triples are {376, 2193, 2225} and {1469, 1647, 2225}. (The Latchkey algorithm also produces two upper non-primitive triples: {975, 2000, 2225} and {2000, 975, 2225} (the Pips for these triples are not shown on the graph)). There are two incongruent Pythagorean triangles with hypotenuse 2225. This example is analogous to Fig. 3.

The second value of r, 2501, is an example from row 8 in Table FSTS. The composite $r_2 = 2501 = r(289) = c(109) = 41 \cdot 61$ with two Pip pairs as shown on the graph; the upper primitive triples are {100, 2499, 2501} and {980, 2301, 2501}. There are two incongruent Pythagorean triangles with hypotenuse 2501. This example is analogous to Fig. 3.

The third value of r, 2665, is an example from row 13 in Table FSTS. The composite $r_3 = 2665 = r(306) = c(117) = 5 \cdot 13 \cdot 41$ corresponds upper primitive triples {73, 2664, 2665}, {816, 2537, 2665}, {1207, 2376, 2665} and {1824, 1943, 2665}. There are four incongruent Pythagorean triangles with hypotenuse 2665.

The forth value of r, 2809, is an example from row 2 of Table FSTS. Composite $r_4 = 2809 = r(324) = c(123) =$ is 53^2 corresponds to the upper primitive triple {1241, 2520, 2809}. There is one Pythagorean triangle with hypotenuse 2809. This example is analogous to Fig. 1.

The fifth value of r, 2969, is an example from row 1 in Table FSTS. $r_5 = 2969 = r(342) = p(211)$ is a Fermat prime similar to Fig. 1; the upper primitive triple is {231, 2960, 2969}.

F. Fixed |y – x| Primitive Triples Algorithm

Early it was observed that there are primitive triples where |y – x| = 1 throughout the GTS tables. The first example occurs in Tables 1.1 with the classic triple {3, 4, 5}. The next example occurs in Tables 2.2 with {20, 21, 29}. In this section the initial terms of an infinite sequence of triples satisfying the condition |y – x| = 1 is found. It starts with triple {3, 4, 5} and extends to triple {803760, 803761,1136689} where it is halted. The procedure to find terms of this infinite sequence is called Fixed |y – x| Primitive Triples, an algorithm that includes a parallel recurrence sequence as developed below. Also found are triples with fixed |y – x| equal to 7, 17 and 23, the next three values of |y – x| that occur in the GTS tables. The results are reported in Table CPL.

Table CPL is organized into four sectors, one each for the Δ values 1, 7, 17 and 23. These four sectors each contain a sequence of primitive triples that correspond to each value of |y – x| considered. The column heads for all sectors is found at the top just under the table title. Each sector is labeled by its value of |y – x| = Δ as well as its parallel recurrence sequence that applies (not the same in all sectors of the table). Each sector has six columns: *ν*, *m*, *n*, *Primitive Triple*, *ϕ* and *Table*. ν is row number/sequence term number, m is table number, n is sequence term variable, and ϕ is arctan(y/x) (ϕ is the angle of the hypotenuse relative to the x-axis), primitive triple and table number, this last entry being taken from the relevant GTS table. The values of ϕ in all sectors of the table approach 45°.

The first nine rows in sector Δ = |y – x| = 1 are copied from their respective applicable tables with table numbers recorded in column *Table*. The last triple in GTS Tables where |y – x| = 1 is in Table 29.2 where r = 5741. Table 29.2 is generated by TS3 and the triple is {4060, 4059, 5741}. Thus the upper symmetric triple is {4059, 4060, 5741}. Column m_s in Table 29.2 in row 21 reports $m_s = 41$. Thus the upper symmetric triple is in Table 41.1 (this table is not included here but is not required to proceed). Since Table 41.1 is generated by TS1, $r = ½ (n^2 + 41^2) = 5741$. Solving r for n results in n = 99. Entering 99 in column n completes row 10 in the first sector.

To continue the table for |y – x| = 1, it was noticed that there is a co-pattern in columns m and n. These two columns starting at n = 5 form a parallel recurrence sequence where m assumes the same value of n, in this case 5, two rows below where this relationship continues on down the table. This parallel recurrence sequence can be expressed as the relation m(ν + 2) = n(ν), a kind of bootstrapping action. All four sectors in column *Table* were ordered according to their order of occurrence of Δ = |y – x| in the GTS tables. The order chosen was that of increasing table number in each sector. Departure from this ordering would have disrupted the parallel recurrence sequences preventing further analysis. Each sector has its own parallel recurrence sequence.

The parallel recurrence sequence for Δ = 1 places m = 70 in row 11. The next step of the solution is more involved since the primitive triple is unknown unlike the case directly above it. That m = 70 is even implies TS2 governs. First, y(m, n) and x(m, n) can be evaluated by substituting m = 70. For TS2 $y(70, n) = n^2 – 70^2$ and x(70) = 2 · 70 n. Then evaluating |y – x| = 1 by substituting expressions y(m, n) and x(m, n) into y – x using y – x = ±1, then $y(70, n) – x(70, n) = (n^2 – 70^2) – 2 \cdot 70 n = \pm 1$. Solving this equation for the choice y – x = +1 results in solutions n = –29 and n = 169. (The choice y – x = –1 results in quadratic equation with irrational roots, not natural number roots as required). Negative n is not a natural number so that n = 169. Thus n = 169 was entered on row 11. Solving for x results in x = 2 · 70 n = 2 · 70 · 169 = 23660 and y – x = 1 implies y = 23661 The Pythagorean Theorem implies r = 33461. Proceeding on to row 12, the parallel recurrence sequence implies m = 99. The triple in row 12 was assumed to be the symmetric triple of that in row 11 where x and y are swapped. With the new x odd TS1 governs. Solving for n yields 239. The rest of the sectors can were filled out using the above procedures. The values of $\phi = \arctan(y/x)$ quickly approach 45°.

The procedure above was repeated for values of $|y - x|$ of 7, 17 and 23 in Table CPL. Note that the parallel recurrence sequence rule inferred from the initial triples for the last three sectors is $m(\nu + 4) = n(\nu)$. Employment of parts or all of Fixed $|y - x|$ Primitive Triples is required when m > 32 since the largest Table number included in this paper is 32. This includes the values of $|y - x|$ of 1, 17 and 23. Note that the values of r grow more slowly for parallel recurrence sequence $m(\nu + 4) = n(\nu)$. The five primitive triples determined by the parallel recurrence sequence were searched for in Latchkey. Latchkey reproduced these triples with m and n verified. Of the five, 33461 and 10357 are Fermat primes as detected by Latchkey.

There are a number of Fermat composites in the four sectors of Table CPL. They all correspond to two or more symmetric pairs of primitive Pythagorean triples analogous to r = 65. Only one of the primitive triple pairs shows up in Table CPL. The other triple(s) has (have) different x and y so that Δ for those triple(s) differ from 1, 7, 17 or 23. As an example consider the case of Δ = 23. One of the two upper triples is in row 12: {1525, 1548, 2173} where y – x = 23 where $2173 = 41 \cdot 53$. The other upper triple is {715, 2052, 2173}: for this triple Δ = 2052 – 715 = 1337, so {715, 2052, 2173} is not a candidate for sector 23 in Table CPL.

G. Patterns

A central feature of this paper is patterns: there are more than a dozen, most of which are new. Eq. 1 and Eq. 2 are similar patterns in the form of infinite sequences; likewise r – y = 1 and r – y = 2 that accompany these equations are patterns. Staircase incorporates the crucial pattern of the step number s = r – y that provides for the calculation of the primitive triples that are produced by the step 1 triples sequences of Eq. 1 and step 2 triples sequences of Eq. 2. Staircase created new patterns of triples sequences for steps 8, 9 and 18, each similar to each other and to the triples sequences of Eq. 1 and Eq. 2. And Staircase produced triples sequences at every step even though many steps were repetitious consisting entirely of non-primitive triples sequences. Then it was found that the triples patterns of steps 1, 2, 8, 9 and18 could be formed into three similar patterns, TS1, TS2 and TS3 which produce infinitely many triples sequences. Next the $r(\nu_r)$ sequence was constructed by reordering the triples sequences from TS1, TS2 and TS3 in Table RPC1. The $r(\nu_r)$ sequence in Table RPC1 was further modified into two component sequences of Fermat primes and Fermat composites. New patterns were developed systematically in Table RPC2. Yet many more patterns are produced by the transformation of Table RPC1 into Table FSTS where the Fermat systematic triples sequences' were developed. The final set of patterns are the parallel recurrence sequences in the Fixed |y – x| Primitive Triples algorithm which produced distinct patterns of triples. The patterns identified above are consistent with Keith Devlin's theme is his book *Mathematics: The Science of Patterns*[9].

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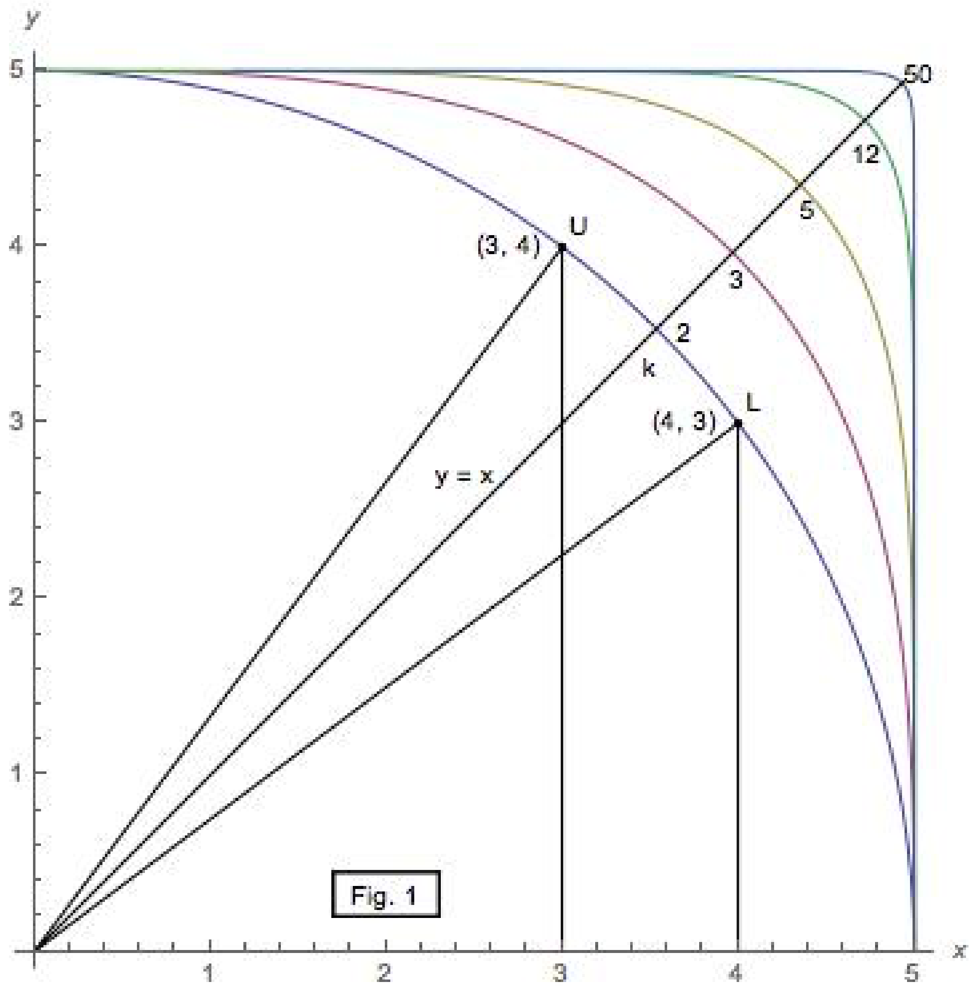

y
5
4
3
2
1
x
1
2
3
4
5
50
12
5
3
2
k
U
(3, 4)
L
(4, 3)
y = x
Fig. 1

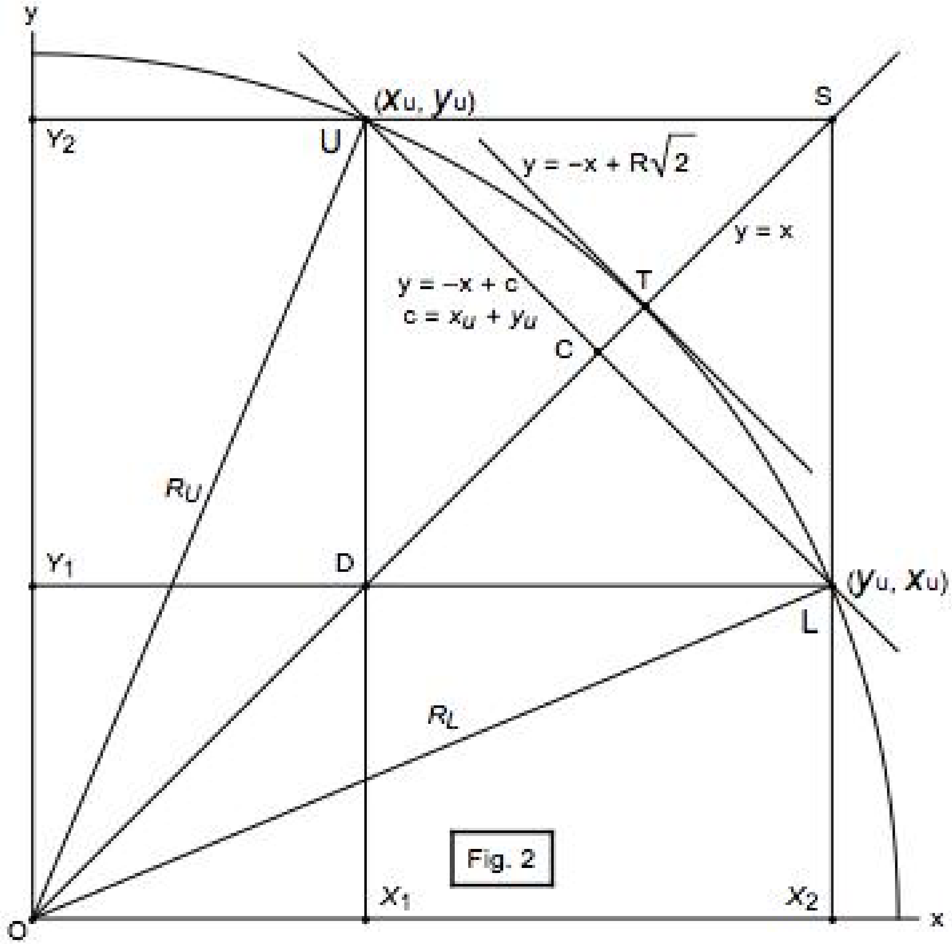
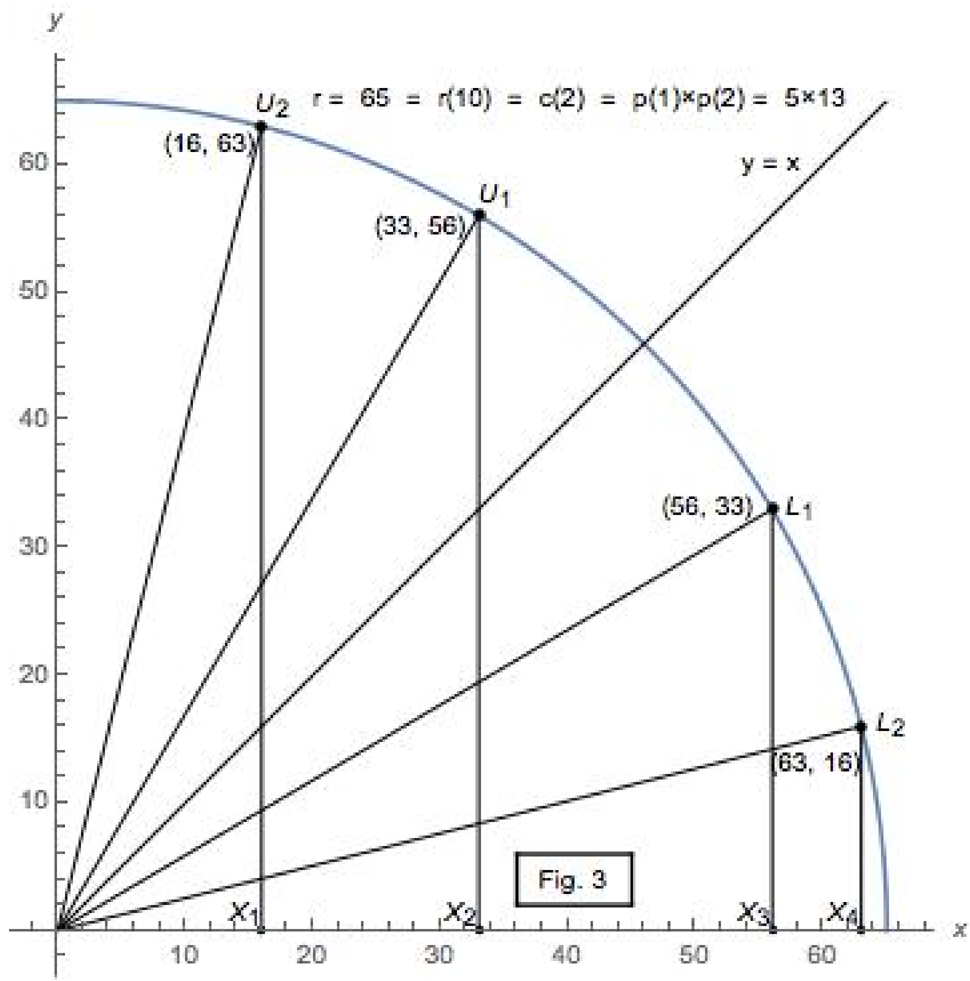


Fig. 2


r = 65 = r(10) = c(2) = p(1)×p(2) = 5×13
y = x
(16, 63)
(33, 56)
(56, 33)
(63, 16)
Fig. 3

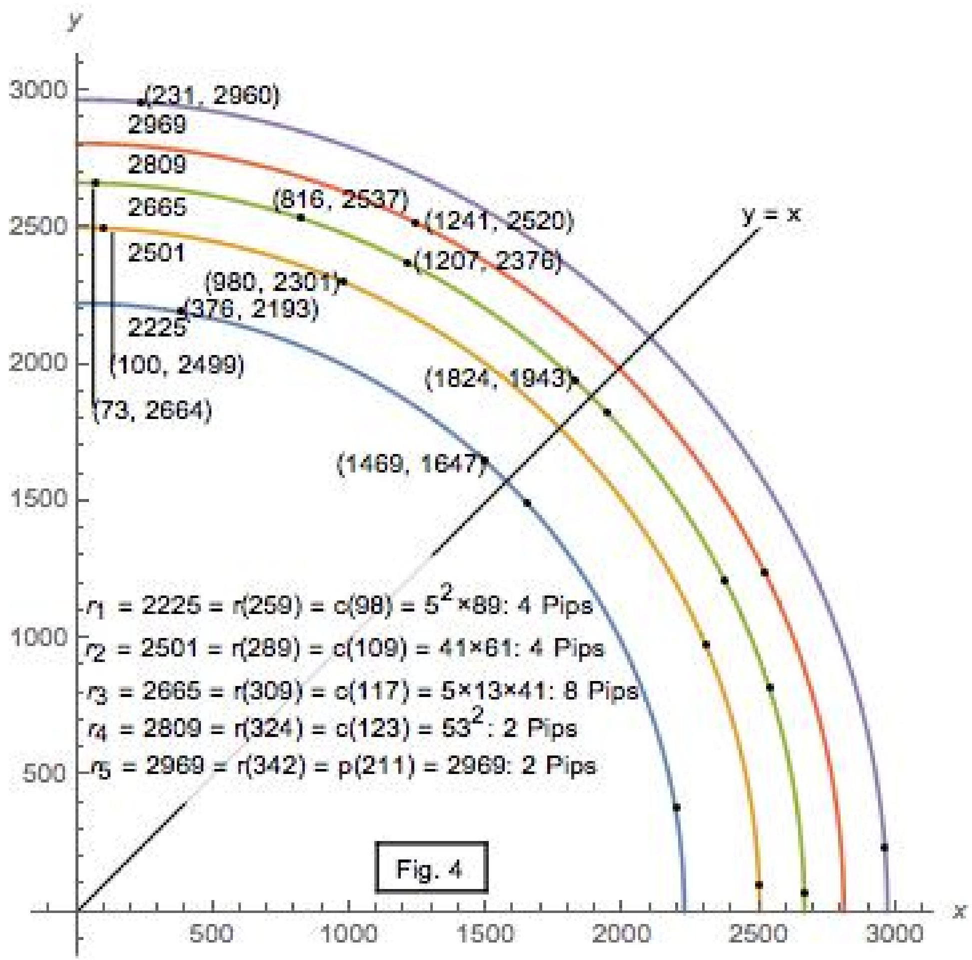

y
x
(231, 2960)
2969
2809
2665
2501
2225
(816, 2537)
(1241, 2520)
y = x
(1207, 2376)
(980, 2301)
(376, 2193)
(100, 2499)
(73, 2664)
(1824, 1943)
(1469, 1647)
r1 = 2225 = r(259) = c(98) = 5^2 ×89: 4 Pips
r2 = 2501 = r(289) = c(109) = 41×61: 4 Pips
r3 = 2665 = r(309) = c(117) = 5×13×41: 8 Pips
r4 = 2809 = r(324) = c(123) = 53^2: 2 Pips
r5 = 2969 = r(342) = p(211) = 2969: 2 Pips
3000
2500
2000
1500
1000
500
0

Fig. 4

Table SA Staircase Search For New Triples Sequences

ν	r^2	Step 1			Step 2			Step 3			Step 8		
1	1	x^2	x	triple									
2	4	3	1.73		x^2	x	triple						
3	9	5	2.24		8	2.83		x^2	x	triple			
4	16	7	2.65		12	3.46		15	3.87				
5	25	9	3	{3, 4, 5}	16	4	{4, 3, 5}	21	4.58				
6	36	11	3.32		20	4.47		27	5.2				
7	49	13	3.61		24	4.9		33	5.74				
8	64	15	3.87		28	5.29		39	6.24		x^2	x	triple
9	81	17	4.12		32	5.66		45	6.71		80	8.94	
10	100	19	4.36		36	6	{6, 8, 10}	51	7.14		96	9.8	
11	121	21	4.58		40	6.32	***{3, 4, 5}***	57	7.55		112	10.6	
12	144	23	4.8		44	6.63		63	7.94		128	11.3	
13	169	25	5	{5, 12, 13}	48	6.93		69	8.31		144	12	{12, 5, 13}
14	196	27	5.2		52	7.21		75	8.66		160	12.6	
15	225	29	5.39		56	7.48		81	9	{9, 12, 15}	176	13.3	
16	256	31	5.57		60	7.75		87	9.33	***{3, 4, 5}***	192	13.9	
17	289	33	5.74		64	8	{8, 15, 17}	93	9.64		208	14.4	
18	324	35	5.92		68	8.25		99	9.95		224	15	
19	361	37	6.08		72	8.49		105	10.2		240	15.5	
20	400	39	6.24		76	8.72		111	10.5		256	16	{16, 12, 20}
21	441	41	6.4		80	8.94		117	10.8		272	16.5	***{4, 3, 5}***
22	484	43	6.56		84	9.17		123	11.1		288	17	
23	529	45	6.71		88	9.38		129	11.4		304	17.4	
24	576	47	6.86		92	9.59		135	11.6		320	17.9	
25	625	49	7	{7, 24, 25}	96	9.8		141	11.9		336	18.3	
26	676	51	7.14		100	10	{10, 24, 26}	147	12.1		352	18.8	
27	729	53	7.28		104	10.2	***{5, 12, 13}***	153	12.4		368	19.2	
28	784	55	7.42		108	10.4		159	12.6		384	19.6	
29	841	57	7.55		112	10.6		165	12.8		400	20	{20, 21, 29}
30	900	59	7.68		116	10.8		171	13.1		416	20.4	
31	961	61	7.81		120	11		177	13.3		432	20.8	
32	1024	63	7.94		124	11.1		183	13.5		448	21.2	
33	1089	65	8.06		128	11.3		189	13.7		464	21.5	
34	1156	67	8.19		132	11.5		195	14		480	21.9	
35	1225	69	8.31		136	11.7		201	14.2		496	22.3	
36	1296	71	8.43		140	11.8		207	14.4		512	22.6	
37	1369	73	8.54		144	12	{12, 35, 37}	213	14.6		528	23	
38	1444	75	8.66		148	12.2		219	14.8		544	23.3	
39	1521	77	8.77		152	12.3		225	15	{15, 36, 39}	560	23.7	
40	1600	79	8.89		156	12.5		231	15.2	***{5, 12, 13}***	576	24	{24, 32, 40}
41	1681	81	9	{9, 40, 41}	160	12.6		237	15.4		592	24.3	***{3, 4, 5}***

Table 1.1 Triples Sequences Step 1: TS1; n(ν)=2ν+1;m=1

ν	n(ν)	x = n	y = (n²- 1)/2	r = (n²+ 1)/2	k = (r-1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y – x
1	3	3	4	5	1		8.13	2	1	1	1	1	1
2	5	5	12	13	3	2	22.38	8	2	1	1	1	7
3	7	7	24	25	6	3	28.74	18	3	1	1	1	17
4	9	9	40	41	10	4	32.32	32	4	1	1	1	31
5	11	11	60	61	15	5	34.61	50	5	1	1	1	49
6	13	13	84	85	21	6	36.20	72	6	1	1	1	71
7	15	15	112	113	28	7	37.37	98	7	1	1	1	97
8	17	17	144	145	36	8	38.27	128	8	1	1	1	127
9	19	19	180	181	45	9	38.98	162	9	1	1	1	161
10	21	21	220	221	55	10	39.55	200	10	1	1	1	199
11	23	23	264	265	66	11	40.02	242	11	1	1	1	241
12	25	25	312	313	78	12	40.42	288	12	1	1	1	287
13	27	27	364	365	91	13	40.76	338	13	1	1	1	337
14	29	29	420	421	105	14	41.05	392	14	1	1	1	391
15	31	31	480	481	120	15	41.31	450	15	1	1	1	449
16	33	33	544	545	136	16	41.53	512	16	1	1	1	511
17	35	35	612	613	153	17	41.73	578	17	1	1	1	577
18	37	37	684	685	171	18	41.91	648	18	1	1	1	647
19	39	39	760	761	190	19	42.06	722	19	1	1	1	721
20	41	41	840	841	210	20	42.21	800	20	1	1	1	799
21	43	43	924	925	231	21	42.34	882	21	1	1	1	881
22	45	45	1012	1013	253	22	42.46	968	22	1	1	1	967
23	47	47	1104	1105	276	23	42.56	1058	23	1	1	1	1057
24	49	49	1200	1201	300	24	42.66	1152	24	1	1	1	1151
25	51	51	1300	1301	325	25	42.76	1250	25	1	1	1	1249
26	53	53	1404	1405	351	26	42.84	1352	26	1	1	1	1351
27	55	55	1512	1513	378	27	42.92	1458	27	1	1	1	1457
28	57	57	1624	1625	406	28	42.99	1568	28	1	1	1	1567
29	59	59	1740	1741	435	29	43.06	1682	29	1	1	1	1681
30	61	61	1860	1861	465	30	43.12	1800	30	1	1	1	1799
31	63	63	1984	1985	496	31	43.18	1922	31	1	1	1	1921
32	65	65	2112	2113	528	32	43.24	2048	32	1	1	1	2047
33	67	67	2244	2245	561	33	43.29	2178	33	1	1	1	2177
34	69	69	2380	2381	595	34	43.34	2312	34	1	1	1	2311
35	71	71	2520	2521	630	35	43.39	2450	35	1	1	1	2449
36	73	73	2664	2665	666	36	43.43	2592	36	1	1	1	2591
37	75	75	2812	2813	703	37	43.47	2738	37	1	1	1	2737
38	77	77	2964	2965	741	38	43.51	2888	38	1	1	1	2887
39	79	79	3120	3121	780	39	43.55	3042	39	1	1	1	3041
40	81	81	3280	3281	820	40	43.59	3200	40	1	1	1	3199

Table 1.2 Triples Sequences Step 2: TS3; n(ν)=2ν; m=1

ν	n(ν)	x = 2n	$y = n^2 - 1$	$r = n^2 + 1$	k = (r - 1)/4	Δk	γ	$s^s = r - x$	m_S	α	β	$s_P = (r - y)/\alpha$	y – x
1	2	4	3	5	1		-8.13	1	1	1	1	2	-1
2	4	8	15	17	4	3	16.93	9	3	1	1	2	7
3	6	12	35	37	9	5	26.08	25	5	1	1	2	23
4	8	16	63	65	16	7	30.75	49	7	1	1	2	47
5	10	20	99	101	25	9	33.58	81	9	1	1	2	79
6	12	24	143	145	36	11	35.48	121	11	1	1	2	119
7	14	28	195	197	49	13	36.83	169	13	1	1	2	167
8	16	32	255	257	64	15	37.85	225	15	1	1	2	223
9	18	36	323	325	81	17	38.64	289	17	1	1	2	287
10	20	40	399	401	100	19	39.28	361	19	1	1	2	359
11	22	44	483	485	121	21	39.80	441	21	1	1	2	439
12	24	48	575	577	144	23	40.23	529	23	1	1	2	527
13	26	52	675	677	169	25	40.60	625	25	1	1	2	623
14	28	56	783	785	196	27	40.91	729	27	1	1	2	727
15	30	60	899	901	225	29	41.18	841	29	1	1	2	839
16	32	64	1023	1025	256	31	41.42	961	31	1	1	2	959
17	34	68	1155	1157	289	33	41.63	1089	33	1	1	2	1087
18	36	72	1295	1297	324	35	41.82	1225	35	1	1	2	1223
19	38	76	1443	1445	361	37	41.99	1369	37	1	1	2	1367
20	40	80	1599	1601	400	39	42.14	1521	39	1	1	2	1519
21	42	84	1763	1765	441	41	42.27	1681	41	1	1	2	1679
22	44	88	1935	1937	484	43	42.40	1849	43	1	1	2	1847
23	46	92	2115	2117	529	45	42.51	2025	45	1	1	2	2023
24	48	96	2303	2305	576	47	42.62	2209	47	1	1	2	2207
25	50	100	2499	2501	625	49	42.71	2401	49	1	1	2	2399
26	52	104	2703	2705	676	51	42.80	2601	51	1	1	2	2599
27	54	108	2915	2917	729	53	42.88	2809	53	1	1	2	2807
28	56	112	3135	3137	784	55	42.96	3025	55	1	1	2	3023
29	58	116	3363	3365	841	57	43.03	3249	57	1	1	2	3247
30	60	120	3599	3601	900	59	43.09	3481	59	1	1	2	3479
31	62	124	3843	3845	961	61	43.15	3721	61	1	1	2	3719
32	64	128	4095	4097	1024	63	43.21	3969	63	1	1	2	3967
33	66	132	4355	4357	1089	65	43.27	4225	65	1	1	2	4223
34	68	136	4623	4625	1156	67	43.32	4489	67	1	1	2	4487
35	70	140	4899	4901	1225	69	43.37	4761	69	1	1	2	4759
36	72	144	5183	5185	1296	71	43.41	5041	71	1	1	2	5039
37	74	148	5475	5477	1369	73	43.45	5329	73	1	1	2	5327
38	76	152	5775	5777	1444	75	43.49	5625	75	1	1	2	5623
39	78	156	6083	6085	1521	77	43.53	5929	77	1	1	2	5927
40	80	160	6399	6401	1600	79	43.57	6241	79	1	1	2	6239

Table 2 Triples Sequences Step 8: TS2; n(ν)=2ν+1; m=2

ν	n(ν)	x = 4n	y = n^2- 4	r = n^2+ 4	k = (r-1)/4	Δk	γ	s_s= r - x	m_S	α	β	s_p = (r-y)/α	y – x
1	3	12	5	13	3		-22.38	1	1	1	1	8	-7
2	5	20	21	29	7	4	1.40	9	3	1	1	8	1
3	7	28	45	53	13	6	13.11	25	5	1	1	8	17
4	9	36	77	85	21	8	19.94	49	7	1	1	8	41
5	11	44	117	125	31	10	24.39	81	9	1	1	8	73
6	13	52	165	173	43	12	27.51	121	11	1	1	8	113
7	15	60	221	229	57	14	29.81	169	13	1	1	8	161
8	17	68	285	293	73	16	31.58	225	15	1	1	8	217
9	19	76	357	365	91	18	32.98	289	17	1	1	8	281
10	21	84	437	445	111	20	34.12	361	19	1	1	8	353
11	23	92	525	533	133	22	35.06	441	21	1	1	8	433
12	25	100	621	629	157	24	35.85	529	23	1	1	8	521
13	27	108	725	733	183	26	36.53	625	25	1	1	8	617
14	29	116	837	845	211	28	37.11	729	27	1	1	8	721
15	31	124	957	965	241	30	37.62	841	29	1	1	8	833
16	33	132	1085	1093	273	32	38.07	961	31	1	1	8	953
17	35	140	1221	1229	307	34	38.46	1089	33	1	1	8	1081
18	37	148	1365	1373	343	36	38.81	1225	35	1	1	8	1217
19	39	156	1517	1525	381	38	39.13	1369	37	1	1	8	1361
20	41	164	1677	1685	421	40	39.42	1521	39	1	1	8	1513
21	43	172	1845	1853	463	42	39.68	1681	41	1	1	8	1673
22	45	180	2021	2029	507	44	39.91	1849	43	1	1	8	1841
23	47	188	2205	2213	553	46	40.13	2025	45	1	1	8	2017
24	49	196	2397	2405	601	48	40.33	2209	47	1	1	8	2201
25	51	204	2597	2605	651	50	40.51	2401	49	1	1	8	2393
26	53	212	2805	2813	703	52	40.68	2601	51	1	1	8	2593
27	55	220	3021	3029	757	54	40.84	2809	53	1	1	8	2801
28	57	228	3245	3253	813	56	40.98	3025	55	1	1	8	3017
29	59	236	3477	3485	871	58	41.12	3249	57	1	1	8	3241
30	61	244	3717	3725	931	60	41.25	3481	59	1	1	8	3473
31	63	252	3965	3973	993	62	41.37	3721	61	1	1	8	3713
32	65	260	4221	4229	1057	64	41.48	3969	63	1	1	8	3961
33	67	268	4485	4493	1123	66	41.58	4225	65	1	1	8	4217
34	69	276	4757	4765	1191	68	41.68	4489	67	1	1	8	4481
35	71	284	5037	5045	1261	70	41.78	4761	69	1	1	8	4753
36	73	292	5325	5333	1333	72	41.86	5041	71	1	1	8	5033
37	75	300	5621	5629	1407	74	41.95	5329	73	1	1	8	5321
38	77	308	5925	5933	1483	76	42.03	5625	75	1	1	8	5617
39	79	316	6237	6245	1561	78	42.10	5929	77	1	1	8	5921
40	81	324	6557	6565	1641	80	42.17	6241	79	1	1	8	6233

Table 3.1 Triples Sequences Step 9: TS1; n(ν)=2ν+3; m=3

ν	n(ν)	x = 3n	y = $(n^2-9)/2$	r = $(n^2+9)/2$	k = (r -1)/4	Δk	γ	s_s= r-x	m_s	α	β	s_p = (r-y)/α	y – x
1	5	15	8	17	4		-16.93	2	1	1	1	9	-7
2	7	**21**	**20**	**29**	7	3	**-1.40**	8	2	1	1	9	-1
3	9	27	36	45	11	4	8.13	18	3	9	3	1	9
4	11	33	56	65	16	5	14.49	32	4	1	1	9	23
5	13	39	80	89	22	6	19.01	50	5	1	1	9	41
6	15	45	108	117	29	7	22.38	72	6	9	3	1	63
7	17	51	140	149	37	8	24.99	98	7	1	1	9	89
8	19	57	176	185	46	9	27.06	128	8	1	1	9	119
9	21	63	216	225	56	10	28.74	162	9	9	3	1	153
10	23	69	260	269	67	11	30.14	200	10	1	1	9	191
11	25	75	308	317	79	12	31.32	242	11	1	1	9	233
12	27	81	360	369	92	13	32.32	288	12	9	3	1	279
13	29	87	416	425	106	14	33.19	338	13	1	1	9	329
14	31	93	476	485	121	15	33.95	392	14	1	1	9	383
15	33	99	540	549	137	16	34.61	450	15	9	3	1	441
16	35	105	608	617	154	17	35.20	512	16	1	1	9	503
17	37	111	680	689	172	18	35.73	578	17	1	1	9	569
18	39	117	756	765	191	19	36.20	648	18	9	3	1	639
19	41	123	836	845	211	20	36.63	722	19	1	1	9	713
20	43	129	920	929	232	21	37.02	800	20	1	1	9	791
21	45	135	1008	1017	254	22	37.37	882	21	9	3	1	873
22	47	141	1100	1109	277	23	37.70	968	22	1	1	9	959
23	49	147	1196	1205	301	24	38.00	1058	23	1	1	9	1049
24	51	153	1296	1305	326	25	38.27	1152	24	9	3	1	1143
25	53	159	1400	1409	352	26	38.52	1250	25	1	1	9	1241
26	55	165	1508	1517	379	27	38.76	1352	26	1	1	9	1343
27	57	171	1620	1629	407	28	38.98	1458	27	9	3	1	1449
28	59	177	1736	1745	436	29	39.18	1568	28	1	1	9	1559
29	61	183	1856	1865	466	30	39.37	1682	29	1	1	9	1673
30	63	189	1980	1989	497	31	39.55	1800	30	9	3	1	1791
31	65	195	2108	2117	529	32	39.72	1922	31	1	1	9	1913
32	67	201	2240	2249	562	33	39.87	2048	32	1	1	9	2039
33	69	207	2376	2385	596	34	40.02	2178	33	9	3	1	2169
34	71	213	2516	2525	631	35	40.16	2312	34	1	1	9	2303
35	73	219	2660	2669	667	36	40.30	2450	35	1	1	9	2441
36	75	225	2808	2817	704	37	40.42	2592	36	9	3	1	2583
37	77	231	2960	2969	742	38	40.54	2738	37	1	1	9	2729
38	79	237	3116	3125	781	39	40.65	2888	38	1	1	9	2879
39	81	243	3276	3285	821	40	40.76	3042	39	9	3	1	3033
40	83	249	3440	3449	862	41	40.86	3200	40	1	1	9	3191

Table 3.2 Triples Sequences Step 18: TS3; n(ν)=2ν+2; m=3

ν	n(ν)	x = 6n	y = n^2 - 9	r = n^2 + 9	k = (r-1)/4	Δk	γ	s_S= r - x	m_S	α	β	s_P= (r-y)/α	y – x
1	4	24	7	25	6		-28.74	1	1	1	1	18	-17
2	6	36	27	45	11	5	-8.13	9	3	9	3	2	-9
3	8	**48**	**55**	**73**	18	7	**3.89**	25	5	1	1	18	7
4	10	60	91	109	27	9	11.60	49	7	1	1	18	31
5	12	72	135	153	38	11	16.93	81	9	9	3	2	63
6	14	84	187	205	51	13	20.81	121	11	1	1	18	103
7	16	96	247	265	66	15	23.76	169	13	1	1	18	151
8	18	108	315	333	83	17	26.08	225	15	9	3	2	207
9	20	120	391	409	102	19	27.94	289	17	1	1	18	271
10	22	132	475	493	123	21	29.47	361	19	1	1	18	343
11	24	144	567	585	146	23	30.75	441	21	9	3	2	423
12	26	156	667	685	171	25	31.84	529	23	1	1	18	511
13	28	168	775	793	198	27	32.77	625	25	1	1	18	607
14	30	180	891	909	227	29	33.58	729	27	9	3	2	711
15	32	192	1015	1033	258	31	34.29	841	29	1	1	18	823
16	34	204	1147	1165	291	33	34.92	961	31	1	1	18	943
17	36	216	1287	1305	326	35	35.48	1089	33	9	3	2	1071
18	38	228	1435	1453	363	37	35.97	1225	35	1	1	18	1207
19	40	240	1591	1609	402	39	36.42	1369	37	1	1	18	1351
20	42	252	1755	1773	443	41	36.83	1521	39	9	3	2	1503
21	44	264	1927	1945	486	43	37.20	1681	41	1	1	18	1663
22	46	276	2107	2125	531	45	37.54	1849	43	1	1	18	1831
23	48	288	2295	2313	578	47	37.85	2025	45	9	3	2	2007
24	50	300	2491	2509	627	49	38.14	2209	47	1	1	18	2191
25	52	312	2695	2713	678	51	38.40	2401	49	1	1	18	2383
26	54	324	2907	2925	731	53	38.64	2601	51	9	3	2	2583
27	56	336	3127	3145	786	55	38.87	2809	53	1	1	18	2791
28	58	348	3355	3373	843	57	39.08	3025	55	1	1	18	3007
29	60	360	3591	3609	902	59	39.28	3249	57	9	3	2	3231
30	62	372	3835	3853	963	61	39.46	3481	59	1	1	18	3463
31	64	384	4087	4105	1026	63	39.63	3721	61	1	1	18	3703
32	66	396	4347	4365	1091	65	39.80	3969	63	9	3	2	3951
33	68	408	4615	4633	1158	67	39.95	4225	65	1	1	18	4207
34	70	420	4891	4909	1227	69	40.09	4489	67	1	1	18	4471
35	72	432	5175	5193	1298	71	40.23	4761	69	9	3	2	4743
36	74	444	5467	5485	1371	73	40.36	5041	71	1	1	18	5023
37	76	456	5767	5785	1446	75	40.48	5329	73	1	1	18	5311
38	78	468	6075	6093	1523	77	40.60	5625	75	9	3	2	5607
39	80	480	6391	6409	1602	79	40.71	5929	77	1	1	18	5911
40	82	492	6715	6733	1683	81	40.81	6241	79	1	1	18	6223

Table 4 Triples Sequences Step 32: TS2; n(ν)=2ν+3; m=2·2

ν	n(ν)	x = 8n	y = n^2- 16	r = n^1 + 16	k = (r-1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r-y)/α	y – x
1	5	40	9	41	10		-32.32	1	1	1	1	32	-31
2	7	56	33	65	16	6	-14.49	9	3	1	1	32	-23
3	9	**72**	**65**	**97**	24	8	**-2.92**	25	5	1	1	32	-7
4	11	88	105	137	34	10	5.04	49	7	1	1	32	17
5	13	104	153	185	46	12	10.80	81	9	1	1	32	49
6	15	120	209	241	60	14	15.14	121	11	1	1	32	89
7	17	136	273	305	76	16	18.52	169	13	1	1	32	137
8	19	152	345	377	94	18	21.22	225	15	1	1	32	193
9	21	168	425	457	114	20	23.43	289	17	1	1	32	257
10	23	184	513	545	136	22	25.27	361	19	1	1	32	329
11	25	200	609	641	160	24	26.82	441	21	1	1	32	409
12	27	216	713	745	186	26	28.15	529	23	1	1	32	497
13	29	232	825	857	214	28	29.30	625	25	1	1	32	593
14	31	248	945	977	244	30	30.30	729	27	1	1	32	697
15	33	264	1073	1105	276	32	31.18	841	29	1	1	32	809
16	35	280	1209	1241	310	34	31.96	961	31	1	1	32	929
17	37	296	1353	1385	346	36	32.66	1089	33	1	1	32	1057
18	39	312	1505	1537	384	38	33.29	1225	35	1	1	32	1193
19	41	328	1665	1697	424	40	33.86	1369	37	1	1	32	1337
20	43	344	1833	1865	466	42	34.37	1521	39	1	1	32	1489
21	45	360	2009	2041	510	44	34.84	1681	41	1	1	32	1649
22	47	376	2193	2225	556	46	35.27	1849	43	1	1	32	1817
23	49	392	2385	2417	604	48	35.67	2025	45	1	1	32	1993
24	51	408	2585	2617	654	50	36.03	2209	47	1	1	32	2177
25	53	424	2793	2825	706	52	36.37	2401	49	1	1	32	2369
26	55	440	3009	3041	760	54	36.68	2601	51	1	1	32	2569
27	57	456	3233	3265	816	56	36.97	2809	53	1	1	32	2777
28	59	472	3465	3497	874	58	37.25	3025	55	1	1	32	2993
29	61	488	3705	3737	934	60	37.50	3249	57	1	1	32	3217
30	63	504	3953	3985	996	62	37.74	3481	59	1	1	32	3449
31	65	520	4209	4241	1060	64	37.96	3721	61	1	1	32	3689
32	67	536	4473	4505	1126	66	38.17	3969	63	1	1	32	3937
33	69	552	4745	4777	1194	68	38.37	4225	65	1	1	32	4193
34	71	568	5025	5057	1264	70	38.55	4489	67	1	1	32	4457
35	73	584	5313	5345	1336	72	38.73	4761	69	1	1	32	4729
36	75	600	5609	5641	1410	74	38.90	5041	71	1	1	32	5009
37	77	616	5913	5945	1486	76	39.06	5329	73	1	1	32	5297
38	79	632	6225	6257	1564	78	39.21	5625	75	1	1	32	5593
39	81	648	6545	6577	1644	80	39.35	5929	77	1	1	32	5897
40	83	664	6873	6905	1726	82	39.48	6241	79	1	1	32	6209

Table 5.1 Triples Sequences Step 25: TS1; $n(\nu)=2\nu+5$; m=5

ν	$n(\nu)$	x = 5n	y = $(n^2 - 25)/2$	r = $(n^2 + 25)/2$	k = (r-1)/4	Δk	γ	s_S = r - x	m_S	α	β	$s_p = (r-y)/\alpha$	y – x
1	7	35	12	37	9		-26.07	2	1	1	1	25	-23
2	9	45	28	53	13	4	-13.11	8	2	1	1	25	-17
3	11	55	48	73	18	5	-3.89	18	3	1	1	25	-7
4	13	**65**	**72**	**97**	24	6	**2.93**	32	4	1	1	25	7
5	15	75	100	125	31	7	8.13	50	5	25	5	1	25
6	17	85	132	157	39	8	12.22	72	6	1	1	25	47
7	19	95	168	193	48	9	15.51	98	7	1	1	25	73
8	21	105	208	233	58	10	18.22	128	8	1	1	25	103
9	23	115	252	277	69	11	20.47	162	9	1	1	25	137
10	25	125	300	325	81	12	22.38	200	10	25	5	1	175
11	27	135	352	377	94	13	24.02	242	11	1	1	25	217
12	29	145	408	433	108	14	25.44	288	12	1	1	25	263
13	31	155	468	493	123	15	26.68	338	13	1	1	25	313
14	33	165	532	557	139	16	27.77	392	14	1	1	25	367
15	35	175	600	625	156	17	28.74	450	15	25	5	1	425
16	37	185	672	697	174	18	29.61	512	16	1	1	25	487
17	39	195	748	773	193	19	30.39	578	17	1	1	25	553
18	41	205	828	853	213	20	31.10	648	18	1	1	25	623
19	43	215	912	937	234	21	31.74	722	19	1	1	25	697
20	45	225	1000	1025	256	22	32.32	800	20	25	5	1	775
21	47	235	1092	1117	279	23	32.86	882	21	1	1	25	857
22	49	245	1188	1213	303	24	33.35	968	22	1	1	25	943
23	51	255	1288	1313	328	25	33.80	1058	23	1	1	25	1033
24	53	265	1392	1417	354	26	34.22	1152	24	1	1	25	1127
25	55	275	1500	1525	381	27	34.61	1250	25	25	5	1	1225
26	57	285	1612	1637	409	28	34.98	1352	26	1	1	25	1327
27	59	295	1728	1753	438	29	35.31	1458	27	1	1	25	1433
28	61	305	1848	1873	468	30	35.63	1568	28	1	1	25	1543
29	63	315	1972	1997	499	31	35.93	1682	29	1	1	25	1657
30	65	325	2100	2125	531	32	36.20	1800	30	25	5	1	1775
31	67	335	2232	2257	564	33	36.47	1922	31	1	1	25	1897
32	69	345	2368	2393	598	34	36.71	2048	32	1	1	25	2023
33	71	355	2508	2533	633	35	36.95	2178	33	1	1	25	2153
34	73	365	2652	2677	669	36	37.17	2312	34	1	1	25	2287
35	75	375	2800	2825	706	37	37.37	2450	35	25	5	1	2425
36	77	385	2952	2977	744	38	37.57	2592	36	1	1	25	2567
37	79	395	3108	3133	783	39	37.76	2738	37	1	1	25	2713
38	81	405	3268	3293	823	40	37.94	2888	38	1	1	25	2863
39	83	415	3432	3457	864	41	38.11	3042	39	1	1	25	3017
40	85	425	3600	3625	906	42	38.27	3200	40	25	5	1	3175

Table 5.2 Triples Sequences Step 50: TS3; n(ν)=2ν+4; m=5

ν	n(ν)	x = 10n	$y = n^2 - 25$	$r = n^2 + 25$	k = (r-1)/4	Δk	γ	$s_s = r - x$	m_s	α	β	$s_p = (r-y)/\alpha$	y – x
1	6	60	11	61	15		-34.61	1	1	1	1	50	-49
2	8	80	39	89	22	7	-19.01	9	3	1	1	50	-41
3	10	100	75	125	31	9	-8.13	25	5	25	5	2	-25
4	12	120	119	169	42	11	-0.24	49	7	1	1	50	-1
5	14	140	171	221	55	13	5.69	81	9	1	1	50	31
6	16	160	231	281	70	15	10.29	121	11	1	1	50	71
7	18	180	299	349	87	17	13.95	169	13	1	1	50	119
8	20	200	375	425	106	19	16.93	225	15	25	5	2	175
9	22	220	459	509	127	21	19.39	289	17	1	1	50	239
10	24	240	551	601	150	23	21.47	361	19	1	1	50	311
11	26	260	651	701	175	25	23.23	441	21	1	1	50	391
12	28	280	759	809	202	27	24.75	529	23	1	1	50	479
13	30	300	875	925	231	29	26.08	625	25	25	5	2	575
14	32	320	999	1049	262	31	27.24	729	27	1	1	50	679
15	34	340	1131	1181	295	33	28.27	841	29	1	1	50	791
16	36	360	1271	1321	330	35	29.19	961	31	1	1	50	911
17	38	380	1419	1469	367	37	30.01	1089	33	1	1	50	1039
18	40	400	1575	1625	406	39	30.75	1225	35	25	5	2	1175
19	42	420	1739	1789	447	41	31.42	1369	37	1	1	50	1319
20	44	440	1911	1961	490	43	32.04	1521	39	1	1	50	1471
21	46	460	2091	2141	535	45	32.60	1681	41	1	1	50	1631
22	48	480	2279	2329	582	47	33.11	1849	43	1	1	50	1799
23	50	500	2475	2525	631	49	33.58	2025	45	25	5	2	1975
24	52	520	2679	2729	682	51	34.02	2209	47	1	1	50	2159
25	54	540	2891	2941	735	53	34.42	2401	49	1	1	50	2351
26	56	560	3111	3161	790	55	34.80	2601	51	1	1	50	2551
27	58	580	3339	3389	847	57	35.15	2809	53	1	1	50	2759
28	60	600	3575	3625	906	59	35.48	3025	55	25	5	2	2975
29	62	620	3819	3869	967	61	35.78	3249	57	1	1	50	3199
30	64	640	4071	4121	1030	63	36.07	3481	59	1	1	50	3431
31	66	660	4331	4381	1095	65	36.34	3721	61	1	1	50	3671
32	68	680	4599	4649	1162	67	36.59	3969	63	1	1	50	3919
33	70	700	4875	4925	1231	69	36.83	4225	65	25	5	2	4175
34	72	720	5159	5209	1302	71	37.06	4489	67	1	1	50	4439
35	74	740	5451	5501	1375	73	37.27	4761	69	1	1	50	4711
36	76	760	5751	5801	1450	75	37.47	5041	71	1	1	50	4991
37	78	780	6059	6109	1527	77	37.67	5329	73	1	1	50	5279
38	80	800	6375	6425	1606	79	37.85	5625	75	25	5	2	5575
39	82	820	6699	6749	1687	81	38.02	5929	77	1	1	50	5879
40	84	840	7031	7081	1770	83	38.19	6241	79	1	1	50	6191

Table 6 Triples Sequences Step 72: TS2; n(ν)=2ν+5; m=2·3

ν	n(ν)	x = 12n	$y=n^2-36$	$r=n^2+36$	k = (r-1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r-y)/α	y – x
1	7	84	13	85	21		-36.20	1	1	1	1	72	-71
2	9	108	45	117	29	8	-22.38	9	3	9	3	8	-63
3	11	132	85	157	39	10	-12.22	25	5	1	1	72	-47
4	13	**156**	**133**	**205**	51	12	**-4.55**	49	7	1	1	72	-23
5	15	180	189	261	65	14	1.40	81	9	9	3	8	9
6	17	204	253	325	81	16	6.12	121	11	1	1	72	49
7	19	228	325	397	99	18	9.95	169	13	1	1	72	97
8	21	252	405	477	119	20	13.11	225	15	9	3	8	153
9	23	276	493	565	141	22	15.76	289	17	1	1	72	217
10	25	300	589	661	165	24	18.01	361	19	1	1	72	289
11	27	324	693	765	191	26	19.94	441	21	9	3	8	369
12	29	348	805	877	219	28	21.62	529	23	1	1	72	457
13	31	372	925	997	249	30	23.09	625	25	1	1	72	553
14	33	396	1053	1125	281	32	24.39	729	27	9	3	8	657
15	35	420	1189	1261	315	34	25.55	841	29	1	1	72	769
16	37	444	1333	1405	351	36	26.58	961	31	1	1	72	889
17	39	468	1485	1557	389	38	27.51	1089	33	9	3	8	1017
18	41	492	1645	1717	429	40	28.35	1225	35	1	1	72	1153
19	43	516	1813	1885	471	42	29.12	1369	37	1	1	72	1297
20	45	540	1989	2061	515	44	29.81	1521	39	9	3	8	1449
21	47	564	2173	2245	561	46	30.45	1681	41	1	1	72	1609
22	49	588	2365	2437	609	48	31.04	1849	43	1	1	72	1777
23	51	612	2565	2637	659	50	31.58	2025	45	9	3	8	1953
24	53	636	2773	2845	711	52	32.08	2209	47	1	1	72	2137
25	55	660	2989	3061	765	54	32.55	2401	49	1	1	72	2329
26	57	684	3213	3285	821	56	32.98	2601	51	9	3	8	2529
27	59	708	3445	3517	879	58	33.39	2809	53	1	1	72	2737
28	61	732	3685	3757	939	60	33.77	3025	55	1	1	72	2953
29	63	756	3933	4005	1001	62	34.12	3249	57	9	3	8	3177
30	65	780	4189	4261	1065	64	34.45	3481	59	1	1	72	3409
31	67	804	4453	4525	1131	66	34.77	3721	61	1	1	72	3649
32	69	828	4725	4797	1199	68	35.06	3969	63	9	3	8	3897
33	71	852	5005	5077	1269	70	35.34	4225	65	1	1	72	4153
34	73	876	5293	5365	1341	72	35.61	4489	67	1	1	72	4417
35	75	900	5589	5661	1415	74	35.85	4761	69	9	3	8	4689
36	77	924	5893	5965	1491	76	36.09	5041	71	1	1	72	4969
37	79	948	6205	6277	1569	78	36.32	5329	73	1	1	72	5257
38	81	972	6525	6597	1649	80	36.53	5625	75	9	3	8	5553
39	83	996	6853	6925	1731	82	36.73	5929	77	1	1	72	5857
40	85	1020	7189	7261	1815	84	36.93	6241	79	1	1	72	6169

Table 7.1 Triples Sequences Step 49: TS1; $n(\nu)=2\nu+7$; m=7

ν	$n(\nu)$	x = 7n	$y = (n^2 - 49)/2$	$r = (n^2 + 49)/2$	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	$s_P = (r-y)/\alpha$	y - x
1	9	63	16	65	16		-30.75	2	1	1	1	49	-47
2	11	77	36	85	21	5	-19.94	8	2	1	1	49	-41
3	13	91	60	109	27	6	-11.60	18	3	1	1	49	-31
4	15	105	88	137	34	7	-5.03	32	4	1	1	49	-17
5	17	**119**	**120**	**169**	42	8	**0.24**	50	5	1	1	49	1
6	19	133	156	205	51	9	4.55	72	6	1	1	49	23
7	21	147	196	245	61	10	8.13	98	7	49	7	1	49
8	23	161	240	289	72	11	11.15	128	8	1	1	49	79
9	25	175	288	337	84	12	13.72	162	9	1	1	49	113
10	27	189	340	389	97	13	15.93	200	10	1	1	49	151
11	29	203	396	445	111	14	17.86	242	11	1	1	49	193
12	31	217	456	505	126	15	19.55	288	12	1	1	49	239
13	33	231	520	569	142	16	21.05	338	13	1	1	49	289
14	35	245	588	637	159	17	22.38	392	14	49	7	1	343
15	37	259	660	709	177	18	23.58	450	15	1	1	49	401
16	39	273	736	785	196	19	24.65	512	16	1	1	49	463
17	41	287	816	865	216	20	25.62	578	17	1	1	49	529
18	43	301	900	949	237	21	26.51	648	18	1	1	49	599
19	45	315	988	1037	259	22	27.32	722	19	1	1	49	673
20	47	329	1080	1129	282	23	28.06	800	20	1	1	49	751
21	49	343	1176	1225	306	24	28.74	882	21	49	7	1	833
22	51	357	1276	1325	331	25	29.37	968	22	1	1	49	919
23	53	371	1380	1429	357	26	29.95	1058	23	1	1	49	1009
24	55	385	1488	1537	384	27	30.50	1152	24	1	1	49	1103
25	57	399	1600	1649	412	28	31.00	1250	25	1	1	49	1201
26	59	413	1716	1765	441	29	31.47	1352	26	1	1	49	1303
27	61	427	1836	1885	471	30	31.91	1458	27	1	1	49	1409
28	63	441	1960	2009	502	31	32.32	1568	28	49	7	1	1519
29	65	455	2088	2137	534	32	32.71	1682	29	1	1	49	1633
30	67	469	2220	2269	567	33	33.07	1800	30	1	1	49	1751
31	69	483	2356	2405	601	34	33.42	1922	31	1	1	49	1873
32	71	497	2496	2545	636	35	33.74	2048	32	1	1	49	1999
33	73	511	2640	2689	672	36	34.05	2178	33	1	1	49	2129
34	75	525	2788	2837	709	37	34.34	2312	34	1	1	49	2263
35	77	539	2940	2989	747	38	34.61	2450	35	49	7	1	2401
36	79	553	3096	3145	786	39	34.88	2592	36	1	1	49	2543
37	81	567	3256	3305	826	40	35.12	2738	37	1	1	49	2689
38	83	581	3420	3469	867	41	35.36	2888	38	1	1	49	2839
39	85	595	3588	3637	909	42	35.59	3042	39	1	1	49	2993
40	87	609	3760	3809	952	43	35.80	3200	40	1	1	49	3151

Table 7.2 Triples Sequences Step 98: TS3; n(ν)=2ν+6; m=7

ν	n(ν)	x = 14n	y = (n^2- 49)	r = (n^2+ 49)	k = (r - 1)/4	Δk	γ	s_s = r - x	m_S	α	β	s_p = (r - y)/α	y - x
1	8	112	15	113	28		-37.37	1	1	1	1	98	-97
2	10	140	51	149	37	9	-24.98	9	3	1	1	98	-89
3	12	168	95	193	48	11	-15.51	25	5	1	1	98	-73
4	14	196	147	245	61	13	-8.13	49	7	49	7	2	-49
5	16	**224**	**207**	**305**	76	15	**-2.26**	81	9	1	1	98	-17
6	18	252	275	373	93	17	2.50	121	11	1	1	98	23
7	20	280	351	449	112	19	6.42	169	13	1	1	98	71
8	22	308	435	533	133	21	9.70	225	15	1	1	98	127
9	24	336	527	625	156	23	12.48	289	17	1	1	98	191
10	26	364	627	725	181	25	14.86	361	19	1	1	98	263
11	28	392	735	833	208	27	16.93	441	21	49	7	2	343
12	30	420	851	949	237	29	18.73	529	23	1	1	98	431
13	32	448	975	1073	268	31	20.32	625	25	1	1	98	527
14	34	476	1107	1205	301	33	21.73	729	27	1	1	98	631
15	36	504	1247	1345	336	35	22.99	841	29	1	1	98	743
16	38	532	1395	1493	373	37	24.13	961	31	1	1	98	863
17	40	560	1551	1649	412	39	25.15	1089	33	1	1	98	991
18	42	588	1715	1813	453	41	26.08	1225	35	49	7	2	1127
19	44	616	1887	1985	496	43	26.92	1369	37	1	1	98	1271
20	46	644	2067	2165	541	45	27.70	1521	39	1	1	98	1423
21	48	672	2255	2353	588	47	28.41	1681	41	1	1	98	1583
22	50	700	2451	2549	637	49	29.06	1849	43	1	1	98	1751
23	52	728	2655	2753	688	51	29.67	2025	45	1	1	98	1927
24	54	756	2867	2965	741	53	30.23	2209	47	1	1	98	2111
25	56	784	3087	3185	796	55	30.75	2401	49	49	7	2	2303
26	58	812	3315	3413	853	57	31.24	2601	51	1	1	98	2503
27	60	840	3551	3649	912	59	31.69	2809	53	1	1	98	2711
28	62	868	3795	3893	973	61	32.12	3025	55	1	1	98	2927
29	64	896	4047	4145	1036	63	32.52	3249	57	1	1	98	3151
30	66	924	4307	4405	1101	65	32.89	3481	59	1	1	98	3383
31	68	952	4575	4673	1168	67	33.25	3721	61	1	1	98	3623
32	70	980	4851	4949	1237	69	33.58	3969	63	49	7	2	3871
33	72	1008	5135	5233	1308	71	33.90	4225	65	1	1	98	4127
34	74	1036	5427	5525	1381	73	34.19	4489	67	1	1	98	4391
35	76	1064	5727	5825	1456	75	34.48	4761	69	1	1	98	4663
36	78	1092	6035	6133	1533	77	34.75	5041	71	1	1	98	4943
37	80	1120	6351	6449	1612	79	35.00	5329	73	1	1	98	5231
38	82	1148	6675	6773	1693	81	35.24	5625	75	1	1	98	5527
39	84	1176	7007	7105	1776	83	35.48	5929	77	49	7	2	5831
40	86	1204	7347	7445	1861	85	35.70	6241	79	1	1	98	6143

Table 8 Triples Sequences Step 128: TS2; n(ν)=2ν+7; m=2·2·2

ν	n(ν)	x=16n	$y = n^2 - 64$	$r = n^2 + 64$	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_p = (r - y)/α	y - x
1	9	144	17	145	36		-38.27	1	1	1	1	128	-127
2	11	176	57	185	46	10	-27.05	9	3	1	1	128	-119
3	13	208	105	233	58	12	-18.21	25	5	1	1	128	-103
4	15	240	161	289	72	14	-11.14	49	7	1	1	128	-79
5	17	272	225	353	88	16	-5.40	81	9	1	1	128	-47
6	19	**304**	**297**	**425**	106	18	**-0.67**	121	11	1	1	128	-7
7	21	336	377	505	126	20	3.29	169	13	1	1	128	41
8	23	368	465	593	148	22	6.64	225	15	1	1	128	97
9	25	400	561	689	172	24	9.51	289	17	1	1	128	161
10	27	432	665	793	198	26	11.99	361	19	1	1	128	233
11	29	464	777	905	226	28	14.16	441	21	1	1	128	313
12	31	496	897	1025	256	30	16.06	529	23	1	1	128	401
13	33	528	1025	1153	288	32	17.75	625	25	1	1	128	497
14	35	560	1161	1289	322	34	19.25	729	27	1	1	128	601
15	37	592	1305	1433	358	36	20.60	841	29	1	1	128	713
16	39	624	1457	1585	396	38	21.82	961	31	1	1	128	833
17	41	656	1617	1745	436	40	22.92	1089	33	1	1	128	961
18	43	688	1785	1913	478	42	23.92	1225	35	1	1	128	1097
19	45	720	1961	2089	522	44	24.84	1369	37	1	1	128	1241
20	47	752	2145	2273	568	46	25.68	1521	39	1	1	128	1393
21	49	784	2337	2465	616	48	26.46	1681	41	1	1	128	1553
22	51	816	2537	2665	666	50	27.17	1849	43	1	1	128	1721
23	53	848	2745	2873	718	52	27.83	2025	45	1	1	128	1897
24	55	880	2961	3089	772	54	28.45	2209	47	1	1	128	2081
25	57	912	3185	3313	828	56	29.02	2401	49	1	1	128	2273
26	59	944	3417	3545	886	58	29.56	2601	51	1	1	128	2473
27	61	976	3657	3785	946	60	30.06	2809	53	1	1	128	2681
28	63	1008	3905	4033	1008	62	30.53	3025	55	1	1	128	2897
29	65	1040	4161	4289	1072	64	30.97	3249	57	1	1	128	3121
30	67	1072	4425	4553	1138	66	31.38	3481	59	1	1	128	3353
31	69	1104	4697	4825	1206	68	31.78	3721	61	1	1	128	3593
32	71	1136	4977	5105	1276	70	32.14	3969	63	1	1	128	3841
33	73	1168	5265	5393	1348	72	32.49	4225	65	1	1	128	4097
34	75	1200	5561	5689	1422	74	32.83	4489	67	1	1	128	4361
35	77	1232	5865	5993	1498	76	33.14	4761	69	1	1	128	4633
36	79	1264	6177	6305	1576	78	33.44	5041	71	1	1	128	4913
37	81	1296	6497	6625	1656	80	33.72	5329	73	1	1	128	5201
38	83	1328	6825	6953	1738	82	33.99	5625	75	1	1	128	5497
39	85	1360	7161	7289	1822	84	34.25	5929	77	1	1	128	5801
40	87	1392	7505	7633	1908	86	34.49	6241	79	1	1	128	6113

Table 9.1 Triples Sequences Step 81: TS1; n(ν)=2ν+9; m=3·3

ν	n(ν)	x = 9n	y = $(n^2$- 81)/2	r = $(n^2$+ 81)/2	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	11	99	20	101	25		-33.58	2	1	1	1	81	-79
2	13	117	44	125	31	6	-24.39	8	2	1	1	81	-73
3	15	135	72	153	38	7	-16.93	18	3	9	3	9	-63
4	17	153	104	185	46	8	-10.79	32	4	1	1	81	-49
5	19	171	140	221	55	9	-5.69	50	5	1	1	81	-31
6	21	189	180	261	65	10	-1.40	72	6	9	3	9	-9
7	23	**207**	**224**	**305**	76	11	**2.26**	98	7	1	1	81	17
8	25	225	272	353	88	12	5.40	128	8	1	1	81	47
9	27	243	324	405	101	13	8.13	162	9	81	9	1	81
10	29	261	380	461	115	14	10.52	200	10	1	1	81	119
11	31	279	440	521	130	15	12.62	242	11	1	1	81	161
12	33	297	504	585	146	16	14.49	288	12	9	3	9	207
13	35	315	572	653	163	17	16.16	338	13	1	1	81	257
14	37	333	644	725	181	18	17.66	392	14	1	1	81	311
15	39	351	720	801	200	19	19.01	450	15	9	3	9	369
16	41	369	800	881	220	20	20.24	512	16	1	1	81	431
17	43	387	884	965	241	21	21.36	578	17	1	1	81	497
18	45	405	972	1053	263	22	22.38	648	18	81	9	1	567
19	47	423	1064	1145	286	23	23.32	722	19	1	1	81	641
20	49	441	1160	1241	310	24	24.19	800	20	1	1	81	719
21	51	459	1260	1341	335	25	24.99	882	21	9	3	9	801
22	53	477	1364	1445	361	26	25.73	968	22	1	1	81	887
23	55	495	1472	1553	388	27	26.42	1058	23	1	1	81	977
24	57	513	1584	1665	416	28	27.06	1152	24	9	3	9	1071
25	59	531	1700	1781	445	29	27.66	1250	25	1	1	81	1169
26	61	549	1820	1901	475	30	28.22	1352	26	1	1	81	1271
27	63	567	1944	2025	506	31	28.74	1458	27	81	9	1	1377
28	65	585	2072	2153	538	32	29.24	1568	28	1	1	81	1487
29	67	603	2204	2285	571	33	29.70	1682	29	1	1	81	1601
30	69	621	2340	2421	605	34	30.14	1800	30	9	3	9	1719
31	71	639	2480	2561	640	35	30.55	1922	31	1	1	81	1841
32	73	657	2624	2705	676	36	30.95	2048	32	1	1	81	1967
33	75	675	2772	2853	713	37	31.32	2178	33	9	3	9	2097
34	77	693	2924	3005	751	38	31.67	2312	34	1	1	81	2231
35	79	711	3080	3161	790	39	32.00	2450	35	1	1	81	2369
36	81	729	3240	3321	830	40	32.32	2592	36	81	9	1	2511
37	83	747	3404	3485	871	41	32.63	2738	37	1	1	81	2657
38	85	765	3572	3653	913	42	32.91	2888	38	1	1	81	2807
39	87	783	3744	3825	956	43	33.19	3042	39	9	3	9	2961
40	89	801	3920	4001	1000	44	33.45	3200	40	1	1	81	3119

Table 9.2 Triples Sequences Step 162: TS3; $n(\nu)=2\nu+8$; $m=3\cdot3$

ν	$n(\nu)$	$x = 18n$	$y = n^2 - 81$	$r = n^2 + 81$	$k = (r - 1)/4$	Δk	γ	$s_S = r - x$	m_S	α	β	$s_P = (r - y)/\alpha$	y - x
1	10	180	19	181	45		-38.97	1	1	1	1	162	-161
2	12	216	63	225	56	11	-28.74	9	3	9	3	18	-153
3	14	252	115	277	69	13	-20.47	25	5	1	1	162	-137
4	16	288	175	337	84	15	-13.71	49	7	1	1	162	-113
5	18	324	243	405	101	17	-8.13	81	9	81	9	2	-81
6	20	360	319	481	120	19	-3.45	121	11	1	1	162	-41
7	22	**396**	**403**	**565**	141	21	**0.50**	169	13	1	1	162	7
8	24	432	495	657	164	23	3.89	225	15	9	3	18	63
9	26	468	595	757	189	25	6.81	289	17	1	1	162	127
10	28	504	703	865	216	27	9.36	361	19	1	1	162	199
11	30	540	819	981	245	29	11.60	441	21	9	3	18	279
12	32	576	943	1105	276	31	13.58	529	23	1	1	162	367
13	34	612	1075	1237	309	33	15.35	625	25	1	1	162	463
14	36	648	1215	1377	344	35	16.93	729	27	81	9	2	567
15	38	684	1363	1525	381	37	18.35	841	29	1	1	162	679
16	40	720	1519	1681	420	39	19.64	961	31	1	1	162	799
17	42	756	1683	1845	461	41	20.81	1089	33	9	3	18	927
18	44	792	1855	2017	504	43	21.88	1225	35	1	1	162	1063
19	46	828	2035	2197	549	45	22.86	1369	37	1	1	162	1207
20	48	864	2223	2385	596	47	23.76	1521	39	9	3	18	1359
21	50	900	2419	2581	645	49	24.59	1681	41	1	1	162	1519
22	52	936	2623	2785	696	51	25.36	1849	43	1	1	162	1687
23	54	972	2835	2997	749	53	26.08	2025	45	81	9	2	1863
24	56	1008	3055	3217	804	55	26.74	2209	47	1	1	162	2047
25	58	1044	3283	3445	861	57	27.36	2401	49	1	1	162	2239
26	60	1080	3519	3681	920	59	27.94	2601	51	9	3	18	2439
27	62	1116	3763	3925	981	61	28.48	2809	53	1	1	162	2647
28	64	1152	4015	4177	1044	63	28.99	3025	55	1	1	162	2863
29	66	1188	4275	4437	1109	65	29.47	3249	57	9	3	18	3087
30	68	1224	4543	4705	1176	67	29.92	3481	59	1	1	162	3319
31	70	1260	4819	4981	1245	69	30.35	3721	61	1	1	162	3559
32	72	1296	5103	5265	1316	71	30.75	3969	63	81	9	2	3807
33	74	1332	5395	5557	1389	73	31.13	4225	65	1	1	162	4063
34	76	1368	5695	5857	1464	75	31.50	4489	67	1	1	162	4327
35	78	1404	6003	6165	1541	77	31.84	4761	69	9	3	18	4599
36	80	1440	6319	6481	1620	79	32.16	5041	71	1	1	162	4879
37	82	1476	6643	6805	1701	81	32.48	5329	73	1	1	162	5167
38	84	1512	6975	7137	1784	83	32.77	5625	75	9	3	18	5463
39	86	1548	7315	7477	1869	85	33.05	5929	77	1	1	162	5767
40	88	1584	7663	7825	1956	87	33.32	6241	79	1	1	162	6079

Table 10 Triples Sequences Step 200: TS2; $n(\nu)=2\nu+9$; $m=2\cdot5$

ν	$n(\nu)$	$x = 20n$	$y = n^2 - 100$	$r = n^2 + 100$	$k = (r - 1)/4$	Δk	γ	$s_s = r - x$	m_s	α	β	$s_p = (r - y)/\alpha$	y - x
1	11	220	21	221	55		-39.55	1	1	1	1	200	-199
2	13	260	69	269	67	12	-30.14	9	3	1	1	200	-191
3	15	300	125	325	81	14	-22.38	25	5	25	5	8	-175
4	17	340	189	389	97	16	-15.93	49	7	1	1	200	-151
5	19	380	261	461	115	18	-10.52	81	9	1	1	200	-119
6	21	420	341	541	135	20	-5.93	121	11	1	1	200	-79
7	23	**460**	**429**	**629**	157	22	**-2.00**	169	13	1	1	200	-31
8	25	500	525	725	181	24	1.40	225	15	25	5	8	25
9	27	540	629	829	207	26	4.36	289	17	1	1	200	89
10	29	580	741	941	235	28	6.95	361	19	1	1	200	161
11	31	620	861	1061	265	30	9.24	441	21	1	1	200	241
12	33	660	989	1189	297	32	11.28	529	23	1	1	200	329
13	35	700	1125	1325	331	34	13.11	625	25	25	5	8	425
14	37	740	1269	1469	367	36	14.75	729	27	1	1	200	529
15	39	780	1421	1621	405	38	16.24	841	29	1	1	200	641
16	41	820	1581	1781	445	40	17.59	961	31	1	1	200	761
17	43	860	1749	1949	487	42	18.82	1089	33	1	1	200	889
18	45	900	1925	2125	531	44	19.94	1225	35	25	5	8	1025
19	47	940	2109	2309	577	46	20.98	1369	37	1	1	200	1169
20	49	980	2301	2501	625	48	21.93	1521	39	1	1	200	1321
21	51	1020	2501	2701	675	50	22.81	1681	41	1	1	200	1481
22	53	1060	2709	2909	727	52	23.63	1849	43	1	1	200	1649
23	55	1100	2925	3125	781	54	24.39	2025	45	25	5	8	1825
24	57	1140	3149	3349	837	56	25.10	2209	47	1	1	200	2009
25	59	1180	3381	3581	895	58	25.76	2401	49	1	1	200	2201
26	61	1220	3621	3821	955	60	26.38	2601	51	1	1	200	2401
27	63	1260	3869	4069	1017	62	26.96	2809	53	1	1	200	2609
28	65	1300	4125	4325	1081	64	27.51	3025	55	25	5	8	2825
29	67	1340	4389	4589	1147	66	28.02	3249	57	1	1	200	3049
30	69	1380	4661	4861	1215	68	28.51	3481	59	1	1	200	3281
31	71	1420	4941	5141	1285	70	28.97	3721	61	1	1	200	3521
32	73	1460	5229	5429	1357	72	29.40	3969	63	1	1	200	3769
33	75	1500	5525	5725	1431	74	29.81	4225	65	25	5	8	4025
34	77	1540	5829	6029	1507	76	30.20	4489	67	1	1	200	4289
35	79	1580	6141	6341	1585	78	30.57	4761	69	1	1	200	4561
36	81	1620	6461	6661	1665	80	30.93	5041	71	1	1	200	4841
37	83	1660	6789	6989	1747	82	31.26	5329	73	1	1	200	5129
38	85	1700	7125	7325	1831	84	31.58	5625	75	25	5	8	5425
39	87	1740	7469	7669	1917	86	31.89	5929	77	1	1	200	5729
40	89	1780	7821	8021	2005	88	32.18	6241	79	1	1	200	6041

Table 11.1 Triples Sequences Step 121: TS1; n(ν)=2ν+11; m=11

ν	n(ν)	x = 11n	y = $(n^2 - 121)/2$	r = $(n^2 + 121)/2$	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	13	143	24	145	36		-35.47	2	1	1	1	121	-119
2	15	165	52	173	43	7	-27.51	8	2	1	1	121	-113
3	17	187	84	205	51	8	-20.81	18	3	1	1	121	-103
4	19	209	120	241	60	9	-15.14	32	4	1	1	121	-89
5	21	231	160	281	70	10	-10.29	50	5	1	1	121	-71
6	23	253	204	325	81	11	-6.12	72	6	1	1	121	-49
7	25	275	252	373	93	12	-2.50	98	7	1	1	121	-23
8	27	**297**	**304**	**425**	106	13	**0.67**	128	8	1	1	121	7
9	29	319	360	481	120	14	3.46	162	9	1	1	121	41
10	31	341	420	541	135	15	5.93	200	10	1	1	121	79
11	33	363	484	605	151	16	8.13	242	11	121	11	1	121
12	35	385	552	673	168	17	10.11	288	12	1	1	121	167
13	37	407	624	745	186	18	11.89	338	13	1	1	121	217
14	39	429	700	821	205	19	13.50	392	14	1	1	121	271
15	41	451	780	901	225	20	14.97	450	15	1	1	121	329
16	43	473	864	985	246	21	16.30	512	16	1	1	121	391
17	45	495	952	1073	268	22	17.53	578	17	1	1	121	457
18	47	517	1044	1165	291	23	18.66	648	18	1	1	121	527
19	49	539	1140	1261	315	24	19.70	722	19	1	1	121	601
20	51	561	1240	1361	340	25	20.66	800	20	1	1	121	679
21	53	583	1344	1465	366	26	21.55	882	21	1	1	121	761
22	55	605	1452	1573	393	27	22.38	968	22	121	11	1	847
23	57	627	1564	1685	421	28	23.16	1058	23	1	1	121	937
24	59	649	1680	1801	450	29	23.88	1152	24	1	1	121	1031
25	61	671	1800	1921	480	30	24.56	1250	25	1	1	121	1129
26	63	693	1924	2045	511	31	25.19	1352	26	1	1	121	1231
27	65	715	2052	2173	543	32	25.79	1458	27	1	1	121	1337
28	67	737	2184	2305	576	33	26.35	1568	28	1	1	121	1447
29	69	759	2320	2441	610	34	26.89	1682	29	1	1	121	1561
30	71	781	2460	2581	645	35	27.39	1800	30	1	1	121	1679
31	73	803	2604	2725	681	36	27.86	1922	31	1	1	121	1801
32	75	825	2752	2873	718	37	28.31	2048	32	1	1	121	1927
33	77	847	2904	3025	756	38	28.74	2178	33	121	11	1	2057
34	79	869	3060	3181	795	39	29.15	2312	34	1	1	121	2191
35	81	891	3220	3341	835	40	29.54	2450	35	1	1	121	2329
36	83	913	3384	3505	876	41	29.90	2592	36	1	1	121	2471
37	85	935	3552	3673	918	42	30.25	2738	37	1	1	121	2617
38	87	957	3724	3845	961	43	30.59	2888	38	1	1	121	2767
39	89	979	3900	4021	1005	44	30.91	3042	39	1	1	121	2921
40	91	1001	4080	4201	1050	45	31.22	3200	40	1	1	121	3079

Table 11.2 Triples Sequences Step 242: TS3; n(ν)=2ν+10; m=11

ν	n(ν)	x = 22n	y = n^2- 121	r = n^2+ 121	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	12	264	23	265	66		-40.02	1	1	1	1	242	-241
2	14	308	75	317	79	13	-31.31	9	3	1	1	242	-233
3	16	352	135	377	94	15	-24.02	25	5	1	1	242	-217
4	18	396	203	445	111	17	-17.86	49	7	1	1	242	-193
5	20	440	279	521	130	19	-12.62	81	9	1	1	242	-161
6	22	484	363	605	151	21	-8.13	121	11	121	11	2	-121
7	24	528	455	697	174	23	-4.25	169	13	1	1	242	-73
8	26	**572**	**555**	**797**	199	25	**-0.86**	225	15	1	1	242	-17
9	28	616	663	905	226	27	2.11	289	17	1	1	242	47
10	30	660	779	1021	255	29	4.73	361	19	1	1	242	119
11	32	704	903	1145	286	31	7.06	441	21	1	1	242	199
12	34	748	1035	1277	319	33	9.15	529	23	1	1	242	287
13	36	792	1175	1417	354	35	11.02	625	25	1	1	242	383
14	38	836	1323	1565	391	37	12.71	729	27	1	1	242	487
15	40	880	1479	1721	430	39	14.25	841	29	1	1	242	599
16	42	924	1643	1885	471	41	15.65	961	31	1	1	242	719
17	44	968	1815	2057	514	43	16.93	1089	33	121	11	2	847
18	46	1012	1995	2237	559	45	18.10	1225	35	1	1	242	983
19	48	1056	2183	2425	606	47	19.19	1369	37	1	1	242	1127
20	50	1100	2379	2621	655	49	20.19	1521	39	1	1	242	1279
21	52	1144	2583	2825	706	51	21.11	1681	41	1	1	242	1439
22	54	1188	2795	3037	759	53	21.97	1849	43	1	1	242	1607
23	56	1232	3015	3257	814	55	22.78	2025	45	1	1	242	1783
24	58	1276	3243	3485	871	57	23.52	2209	47	1	1	242	1967
25	60	1320	3479	3721	930	59	24.22	2401	49	1	1	242	2159
26	62	1364	3723	3965	991	61	24.88	2601	51	1	1	242	2359
27	64	1408	3975	4217	1054	63	25.50	2809	53	1	1	242	2567
28	66	1452	4235	4477	1119	65	26.08	3025	55	121	11	2	2783
29	68	1496	4503	4745	1186	67	26.62	3249	57	1	1	242	3007
30	70	1540	4779	5021	1255	69	27.14	3481	59	1	1	242	3239
31	72	1584	5063	5305	1326	71	27.63	3721	61	1	1	242	3479
32	74	1628	5355	5597	1399	73	28.09	3969	63	1	1	242	3727
33	76	1672	5655	5897	1474	75	28.53	4225	65	1	1	242	3983
34	78	1716	5963	6205	1551	77	28.95	4489	67	1	1	242	4247
35	80	1760	6279	6521	1630	79	29.34	4761	69	1	1	242	4519
36	82	1804	6603	6845	1711	81	29.72	5041	71	1	1	242	4799
37	84	1848	6935	7177	1794	83	30.08	5329	73	1	1	242	5087
38	86	1892	7275	7517	1879	85	30.42	5625	75	1	1	242	5383
39	88	1936	7623	7865	1966	87	30.75	5929	77	121	11	2	5687
40	90	1980	7979	8221	2055	89	31.07	6241	79	1	1	242	5999

Table 12 Triples Sequences Step 288: TS2; n(ν)=2ν+11; m=2·2·3

ν	n(ν)	x = 24n	y = n^2- 144	r = n^2+ 144	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	13	312	25	313	78		-40.42	1	1	1	1	288	-287
2	15	360	81	369	92	14	-32.32	9	3	9	3	32	-279
3	17	408	145	433	108	16	-25.43	25	5	1	1	288	-263
4	19	456	217	505	126	18	-19.55	49	7	1	1	288	-239
5	21	504	297	585	146	20	-14.49	81	9	9	3	32	-207
6	23	552	385	673	168	22	-10.10	121	11	1	1	288	-167
7	25	600	481	769	192	24	-6.28	169	13	1	1	288	-119
8	27	648	585	873	218	26	-2.92	225	15	9	3	32	-63
9	29	**696**	**697**	**985**	246	28	**0.04**	289	17	1	1	288	1
10	31	744	817	1105	276	30	2.68	361	19	1	1	288	73
11	33	792	945	1233	308	32	5.04	441	21	9	3	32	153
12	35	840	1081	1369	342	34	7.15	529	23	1	1	288	241
13	37	888	1225	1513	378	36	9.06	625	25	1	1	288	337
14	39	936	1377	1665	416	38	10.80	729	27	9	3	32	441
15	41	984	1537	1825	456	40	12.37	841	29	1	1	288	553
16	43	1032	1705	1993	498	42	13.82	961	31	1	1	288	673
17	45	1080	1881	2169	542	44	15.14	1089	33	9	3	32	801
18	47	1128	2065	2353	588	46	16.36	1225	35	1	1	288	937
19	49	1176	2257	2545	636	48	17.48	1369	37	1	1	288	1081
20	51	1224	2457	2745	686	50	18.52	1521	39	9	3	32	1233
21	53	1272	2665	2953	738	52	19.49	1681	41	1	1	288	1393
22	55	1320	2881	3169	792	54	20.39	1849	43	1	1	288	1561
23	57	1368	3105	3393	848	56	21.22	2025	45	9	3	32	1737
24	59	1416	3337	3625	906	58	22.01	2209	47	1	1	288	1921
25	61	1464	3577	3865	966	60	22.74	2401	49	1	1	288	2113
26	63	1512	3825	4113	1028	62	23.43	2601	51	9	3	32	2313
27	65	1560	4081	4369	1092	64	24.08	2809	53	1	1	288	2521
28	67	1608	4345	4633	1158	66	24.69	3025	55	1	1	288	2737
29	69	1656	4617	4905	1226	68	25.27	3249	57	9	3	32	2961
30	71	1704	4897	5185	1296	70	25.82	3481	59	1	1	288	3193
31	73	1752	5185	5473	1368	72	26.33	3721	61	1	1	288	3433
32	75	1800	5481	5769	1442	74	26.82	3969	63	9	3	32	3681
33	77	1848	5785	6073	1518	76	27.29	4225	65	1	1	288	3937
34	79	1896	6097	6385	1596	78	27.73	4489	67	1	1	288	4201
35	81	1944	6417	6705	1676	80	28.15	4761	69	9	3	32	4473
36	83	1992	6745	7033	1758	82	28.55	5041	71	1	1	288	4753
37	85	2040	7081	7369	1842	84	28.93	5329	73	1	1	288	5041
38	87	2088	7425	7713	1928	86	29.30	5625	75	9	3	32	5337
39	89	2136	7777	8065	2016	88	29.64	5929	77	1	1	288	5641
40	91	2184	8137	8425	2106	90	29.98	6241	79	1	1	288	5953

Table 13.1 Triples Sequences Step 169: TS1; n(ν)=2ν+13; m=13

ν	n(ν)	x = 13n	y = (n²- 169)/2	r = (n²+ 169)/2	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	15	195	28	197	49		-36.83	2	1	1	1	169	-167
2	17	221	60	229	57	8	-29.81	8	2	1	1	169	-161
3	19	247	96	265	66	9	-23.76	18	3	1	1	169	-151
4	21	273	136	305	76	10	-18.52	32	4	1	1	169	-137
5	23	299	180	349	87	11	-13.95	50	5	1	1	169	-119
6	25	325	228	397	99	12	-9.95	72	6	1	1	169	-97
7	27	351	280	449	112	13	-6.42	98	7	1	1	169	-71
8	29	377	336	505	126	14	-3.29	128	8	1	1	169	-41
9	31	**403**	**396**	**565**	141	15	**-0.50**	162	9	1	1	169	-7
10	33	429	460	629	157	16	2.00	200	10	1	1	169	31
11	35	455	528	697	174	17	4.25	242	11	1	1	169	73
12	37	481	600	769	192	18	6.28	288	12	1	1	169	119
13	39	507	676	845	211	19	8.13	338	13	169	13	1	169
14	41	533	756	925	231	20	9.82	392	14	1	1	169	223
15	43	559	840	1009	252	21	11.36	450	15	1	1	169	281
16	45	585	928	1097	274	22	12.77	512	16	1	1	169	343
17	47	611	1020	1189	297	23	14.08	578	17	1	1	169	409
18	49	637	1116	1285	321	24	15.28	648	18	1	1	169	479
19	51	663	1216	1385	346	25	16.40	722	19	1	1	169	553
20	53	689	1320	1489	372	26	17.44	800	20	1	1	169	631
21	55	715	1428	1597	399	27	18.40	882	21	1	1	169	713
22	57	741	1540	1709	427	28	19.31	968	22	1	1	169	799
23	59	767	1656	1825	456	29	20.15	1058	23	1	1	169	889
24	61	793	1776	1945	486	30	20.94	1152	24	1	1	169	983
25	63	819	1900	2069	517	31	21.68	1250	25	1	1	169	1081
26	65	845	2028	2197	549	32	22.38	1352	26	169	13	1	1183
27	67	871	2160	2329	582	33	23.04	1458	27	1	1	169	1289
28	69	897	2296	2465	616	34	23.66	1568	28	1	1	169	1399
29	71	923	2436	2605	651	35	24.25	1682	29	1	1	169	1513
30	73	949	2580	2749	687	36	24.81	1800	30	1	1	169	1631
31	75	975	2728	2897	724	37	25.33	1922	31	1	1	169	1753
32	77	1001	2880	3049	762	38	25.84	2048	32	1	1	169	1879
33	79	1027	3036	3205	801	39	26.31	2178	33	1	1	169	2009
34	81	1053	3196	3365	841	40	26.77	2312	34	1	1	169	2143
35	83	1079	3360	3529	882	41	27.20	2450	35	1	1	169	2281
36	85	1105	3528	3697	924	42	27.61	2592	36	1	1	169	2423
37	87	1131	3700	3869	967	43	28.01	2738	37	1	1	169	2569
38	89	1157	3876	4045	1011	44	28.38	2888	38	1	1	169	2719
39	91	1183	4056	4225	1056	45	28.74	3042	39	169	13	1	2873
40	93	1209	4240	4409	1102	46	29.09	3200	40	1	1	169	3031

Table 13.2 Triples Sequences Step 338: TS3; n(ν)=2ν+12; m=13

ν	n(ν)	x = 26n	y = n^2- 169	r = n^2+ 169	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	14	364	27	365	91		-40.76	1	1	1	1	338	-337
2	16	416	87	425	106	15	-33.19	9	3	1	1	338	-329
3	18	468	155	493	123	17	-26.67	25	5	1	1	338	-313
4	20	520	231	569	142	19	-21.05	49	7	1	1	338	-289
5	22	572	315	653	163	21	-16.16	81	9	1	1	338	-257
6	24	624	407	745	186	23	-11.88	121	11	1	1	338	-217
7	26	676	507	845	211	25	-8.13	169	13	169	13	2	-169
8	28	728	615	953	238	27	-4.81	225	15	1	1	338	-113
9	30	780	731	1069	267	29	-1.86	289	17	1	1	338	-49
10	32	**832**	**855**	**1193**	298	31	**0.78**	361	19	1	1	338	23
11	34	884	987	1325	331	33	3.15	441	21	1	1	338	103
12	36	936	1127	1465	366	35	5.29	529	23	1	1	338	191
13	38	988	1275	1613	403	37	7.23	625	25	1	1	338	287
14	40	1040	1431	1769	442	39	8.99	729	27	1	1	338	391
15	42	1092	1595	1933	483	41	10.60	841	29	1	1	338	503
16	44	1144	1767	2105	526	43	12.08	961	31	1	1	338	623
17	46	1196	1947	2285	571	45	13.44	1089	33	1	1	338	751
18	48	1248	2135	2473	618	47	14.69	1225	35	1	1	338	887
19	50	1300	2331	2669	667	49	15.85	1369	37	1	1	338	1031
20	52	1352	2535	2873	718	51	16.93	1521	39	169	13	2	1183
21	54	1404	2747	3085	771	53	17.93	1681	41	1	1	338	1343
22	56	1456	2967	3305	826	55	18.86	1849	43	1	1	338	1511
23	58	1508	3195	3533	883	57	19.74	2025	45	1	1	338	1687
24	60	1560	3431	3769	942	59	20.55	2209	47	1	1	338	1871
25	62	1612	3675	4013	1003	61	21.32	2401	49	1	1	338	2063
26	64	1664	3927	4265	1066	63	22.04	2601	51	1	1	338	2263
27	66	1716	4187	4525	1131	65	22.72	2809	53	1	1	338	2471
28	68	1768	4455	4793	1198	67	23.36	3025	55	1	1	338	2687
29	70	1820	4731	5069	1267	69	23.96	3249	57	1	1	338	2911
30	72	1872	5015	5353	1338	71	24.53	3481	59	1	1	338	3143
31	74	1924	5307	5645	1411	73	25.07	3721	61	1	1	338	3383
32	76	1976	5607	5945	1486	75	25.59	3969	63	1	1	338	3631
33	78	2028	5915	6253	1563	77	26.08	4225	65	169	13	2	3887
34	80	2080	6231	6569	1642	79	26.54	4489	67	1	1	338	4151
35	82	2132	6555	6893	1723	81	26.99	4761	69	1	1	338	4423
36	84	2184	6887	7225	1806	83	27.41	5041	71	1	1	338	4703
37	86	2236	7227	7565	1891	85	27.81	5329	73	1	1	338	4991
38	88	2288	7575	7913	1978	87	28.20	5625	75	1	1	338	5287
39	90	2340	7931	8269	2067	89	28.56	5929	77	1	1	338	5591
40	92	2392	8295	8633	2158	91	28.92	6241	79	1	1	338	5903

Table 14 Triples Sequences Step 392 TS2; $n(\nu)=2\nu+13$; $m=2\cdot 7$

ν	$n(\nu)$	$x = 28n$	$y = n^2 - 196$	$r = n^2 + 196$	$k = (r - 1)/4$	Δk	γ	$s_s = r - x$	m_s	α	β	$s_p = (r - y)/\alpha$	$y - x$
1	15	420	29	421	105		-41.05	1	1	1	1	392	-391
2	17	476	93	485	121	16	-33.94	9	3	1	1	392	-383
3	19	532	165	557	139	18	-27.77	25	5	1	1	392	-367
4	21	588	245	637	159	20	-22.38	49	7	49	7	8	-343
5	23	644	333	725	181	22	-17.66	81	9	1	1	392	-311
6	25	700	429	821	205	24	-13.50	121	11	1	1	392	-271
7	27	756	533	925	231	26	-9.81	169	13	1	1	392	-223
8	29	812	645	1037	259	28	-6.54	225	15	1	1	392	-167
9	31	868	765	1157	289	30	-3.61	289	17	1	1	392	-103
10	33	**924**	**893**	**1285**	321	32	**-0.98**	361	19	1	1	392	-31
11	35	980	1029	1421	355	34	1.40	441	21	49	7	8	49
12	37	1036	1173	1565	391	36	3.55	529	23	1	1	392	137
13	39	1092	1325	1717	429	38	5.51	625	25	1	1	392	233
14	41	1148	1485	1877	469	40	7.30	729	27	1	1	392	337
15	43	1204	1653	2045	511	42	8.93	841	29	1	1	392	449
16	45	1260	1829	2221	555	44	10.44	961	31	1	1	392	569
17	47	1316	2013	2405	601	46	11.83	1089	33	1	1	392	697
18	49	1372	2205	2597	649	48	13.11	1225	35	49	7	8	833
19	51	1428	2405	2797	699	50	14.30	1369	37	1	1	392	977
20	53	1484	2613	3005	751	52	15.41	1521	39	1	1	392	1129
21	55	1540	2829	3221	805	54	16.44	1681	41	1	1	392	1289
22	57	1596	3053	3445	861	56	17.40	1849	43	1	1	392	1457
23	59	1652	3285	3677	919	58	18.30	2025	45	1	1	392	1633
24	61	1708	3525	3917	979	60	19.15	2209	47	1	1	392	1817
25	63	1764	3773	4165	1041	62	19.94	2401	49	49	7	8	2009
26	65	1820	4029	4421	1105	64	20.69	2601	51	1	1	392	2209
27	67	1876	4293	4685	1171	66	21.40	2809	53	1	1	392	2417
28	69	1932	4565	4957	1239	68	22.06	3025	55	1	1	392	2633
29	71	1988	4845	5237	1309	70	22.69	3249	57	1	1	392	2857
30	73	2044	5133	5525	1381	72	23.29	3481	59	1	1	392	3089
31	75	2100	5429	5821	1455	74	23.85	3721	61	1	1	392	3329
32	77	2156	5733	6125	1531	76	24.39	3969	63	49	7	8	3577
33	79	2212	6045	6437	1609	78	24.90	4225	65	1	1	392	3833
34	81	2268	6365	6757	1689	80	25.39	4489	67	1	1	392	4097
35	83	2324	6693	7085	1771	82	25.85	4761	69	1	1	392	4369
36	85	2380	7029	7421	1855	84	26.30	5041	71	1	1	392	4649
37	87	2436	7373	7765	1941	86	26.72	5329	73	1	1	392	4937
38	89	2492	7725	8117	2029	88	27.12	5625	75	1	1	392	5233
39	91	2548	8085	8477	2119	90	27.51	5929	77	49	7	8	5537
40	93	2604	8453	8845	2211	92	27.88	6241	79	1	1	392	5849

Table 15.1 Triples Sequences Step 225: TS1; n(ν) = 2ν+15; m=3·5

ν	n(ν)	x = 15n	y = $(n^2 - 255)/2$	r = $(n^2 + 255)/2$	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	17	255	32	257	64		-37.85	2	1	1	1	225	-223
2	19	285	68	293	73	9	-31.58	8	2	1	1	225	-217
3	21	315	108	333	83	10	-26.07	18	3	9	3	25	-207
4	23	345	152	377	94	11	-21.22	32	4	1	1	225	-193
5	25	375	200	425	106	12	-16.93	50	5	25	5	9	-175
6	27	405	252	477	119	13	-13.11	72	6	9	3	25	-153
7	29	435	308	533	133	14	-9.70	98	7	1	1	225	-127
8	31	465	368	593	148	15	-6.64	128	8	1	1	225	-97
9	33	495	432	657	164	16	-3.89	162	9	9	3	25	-63
10	35	525	500	725	181	17	-1.40	200	10	25	5	9	-25
11	37	**555**	**572**	**797**	199	18	**0.87**	242	11	1	1	225	17
12	39	585	648	873	218	19	2.93	288	12	9	3	25	63
13	41	615	728	953	238	20	4.81	338	13	1	1	225	113
14	43	645	812	1037	259	21	6.54	392	14	1	1	225	167
15	45	675	900	1125	281	22	8.13	450	15	225	15	1	225
16	47	705	992	1217	304	23	9.60	512	16	1	1	225	287
17	49	735	1088	1313	328	24	10.96	578	17	1	1	225	353
18	51	765	1188	1413	353	25	12.22	648	18	9	3	25	423
19	53	795	1292	1517	379	26	13.40	722	19	1	1	225	497
20	55	825	1400	1625	406	27	14.49	800	20	25	5	9	575
21	57	855	1512	1737	434	28	15.51	882	21	9	3	25	657
22	59	885	1628	1853	463	29	16.47	968	22	1	1	225	743
23	61	915	1748	1973	493	30	17.37	1058	23	1	1	225	833
24	63	945	1872	2097	524	31	18.22	1152	24	9	3	25	927
25	65	975	2000	2225	556	32	19.01	1250	25	25	5	9	1025
26	67	1005	2132	2357	589	33	19.76	1352	26	1	1	225	1127
27	69	1035	2268	2493	623	34	20.47	1458	27	9	3	25	1233
28	71	1065	2408	2633	658	35	21.14	1568	28	1	1	225	1343
29	73	1095	2552	2777	694	36	21.78	1682	29	1	1	225	1457
30	75	1125	2700	2925	731	37	22.38	1800	30	225	15	1	1575
31	77	1155	2852	3077	769	38	22.96	1922	31	1	1	225	1697
32	79	1185	3008	3233	808	39	23.50	2048	32	1	1	225	1823
33	81	1215	3168	3393	848	40	24.02	2178	33	9	3	25	1953
34	83	1245	3332	3557	889	41	24.51	2312	34	1	1	225	2087
35	85	1275	3500	3725	931	42	24.99	2450	35	25	5	9	2225
36	87	1305	3672	3897	974	43	25.44	2592	36	9	3	25	2367
37	89	1335	3848	4073	1018	44	25.87	2738	37	1	1	225	2513
38	91	1365	4028	4253	1063	45	26.28	2888	38	1	1	225	2663
39	93	1395	4212	4437	1109	46	26.68	3042	39	9	3	25	2817
40	95	1425	4400	4625	1156	47	27.06	3200	40	25	5	9	2975

Table 15.2 Triples Sequences Step 450: TS3; n(ν)=2ν+14; m=3·5

ν	n(ν)	x = 30n	y = n^2- 225	r = n^2+ 225	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	16	480	31	481	120		-41.30	1	1	1	1	450	-449
2	18	540	99	549	137	17	-34.61	9	3	9	3	50	-441
3	20	600	175	625	156	19	-28.74	25	5	25	5	18	-425
4	22	660	259	709	177	21	-23.57	49	7	1	1	450	-401
5	24	720	351	801	200	23	-19.01	81	9	9	3	50	-369
6	26	780	451	901	225	25	-14.96	121	11	1	1	450	-329
7	28	840	559	1009	252	27	-11.36	169	13	1	1	450	-281
8	30	900	675	1125	281	29	-8.13	225	15	225	15	2	-225
9	32	960	799	1249	312	31	-5.23	289	17	1	1	450	-161
10	34	1020	931	1381	345	33	-2.61	361	19	1	1	450	-89
11	36	1080	1071	1521	380	35	-0.24	441	21	9	3	50	-9
12	38	**1140**	**1219**	**1669**	417	37	**1.92**	529	23	1	1	450	79
13	40	1200	1375	1825	456	39	3.89	625	25	25	5	18	175
14	42	1260	1539	1989	497	41	5.69	729	27	9	3	50	279
15	44	1320	1711	2161	540	43	7.35	841	29	1	1	450	391
16	46	1380	1891	2341	585	45	8.88	961	31	1	1	450	511
17	48	1440	2079	2529	632	47	10.29	1089	33	9	3	50	639
18	50	1500	2275	2725	681	49	11.60	1225	35	25	5	18	775
19	52	1560	2479	2929	732	51	12.82	1369	37	1	1	450	919
20	54	1620	2691	3141	785	53	13.95	1521	39	9	3	50	1071
21	56	1680	2911	3361	840	55	15.01	1681	41	1	1	450	1231
22	58	1740	3139	3589	897	57	16.00	1849	43	1	1	450	1399
23	60	1800	3375	3825	956	59	16.93	2025	45	225	15	2	1575
24	62	1860	3619	4069	1017	61	17.80	2209	47	1	1	450	1759
25	64	1920	3871	4321	1080	63	18.62	2401	49	1	1	450	1951
26	66	1980	4131	4581	1145	65	19.39	2601	51	9	3	50	2151
27	68	2040	4399	4849	1212	67	20.12	2809	53	1	1	450	2359
28	70	2100	4675	5125	1281	69	20.81	3025	55	25	5	18	2575
29	72	2160	4959	5409	1352	71	21.47	3249	57	9	3	50	2799
30	74	2220	5251	5701	1425	73	22.08	3481	59	1	1	450	3031
31	76	2280	5551	6001	1500	75	22.67	3721	61	1	1	450	3271
32	78	2340	5859	6309	1577	77	23.23	3969	63	9	3	50	3519
33	80	2400	6175	6625	1656	79	23.76	4225	65	25	5	18	3775
34	82	2460	6499	6949	1737	81	24.27	4489	67	1	1	450	4039
35	84	2520	6831	7281	1820	83	24.75	4761	69	9	3	50	4311
36	86	2580	7171	7621	1905	85	25.21	5041	71	1	1	450	4591
37	88	2640	7519	7969	1992	87	25.66	5329	73	1	1	450	4879
38	90	2700	7875	8325	2081	89	26.08	5625	75	225	15	2	5175
39	92	2760	8239	8689	2172	91	26.48	5929	77	1	1	450	5479
40	94	2820	8611	9061	2265	93	26.87	6241	79	1	1	450	5791

Table 16 Triples Sequences Step 512: TS2; n(ν)=2ν+15; m=2·2·2·2

ν	n(ν)	x = 32n	y = n^2 - 256	r = n^2 + 256	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	17	544	33	545	136		-41.53	1	1	1	1	512	-511
2	19	608	105	617	154	18	-35.20	9	3	1	1	512	-503
3	21	672	185	697	174	20	-29.61	25	5	1	1	512	-487
4	23	736	273	785	196	22	-24.65	49	7	1	1	512	-463
5	25	800	369	881	220	24	-20.24	81	9	1	1	512	-431
6	27	864	473	985	246	26	-16.30	121	11	1	1	512	-391
7	29	928	585	1097	274	28	-12.77	169	13	1	1	512	-343
8	31	992	705	1217	304	30	-9.60	225	15	1	1	512	-287
9	33	1056	833	1345	336	32	-6.73	289	17	1	1	512	-223
10	35	1120	969	1481	370	34	-4.13	361	19	1	1	512	-151
11	37	1184	1113	1625	406	36	-1.77	441	21	1	1	512	-71
12	39	**1248**	**1265**	**1777**	444	38	**0.39**	529	23	1	1	512	17
13	41	1312	1425	1937	484	40	2.37	625	25	1	1	512	113
14	43	1376	1593	2105	526	42	4.18	729	27	1	1	512	217
15	45	1440	1769	2281	570	44	5.86	841	29	1	1	512	329
16	47	1504	1953	2465	616	46	7.40	961	31	1	1	512	449
17	49	1568	2145	2657	664	48	8.83	1089	33	1	1	512	577
18	51	1632	2345	2857	714	50	10.17	1225	35	1	1	512	713
19	53	1696	2553	3065	766	52	11.40	1369	37	1	1	512	857
20	55	1760	2769	3281	820	54	12.56	1521	39	1	1	512	1009
21	57	1824	2993	3505	876	56	13.64	1681	41	1	1	512	1169
22	59	1888	3225	3737	934	58	14.66	1849	43	1	1	512	1337
23	61	1952	3465	3977	994	60	15.61	2025	45	1	1	512	1513
24	63	2016	3713	4225	1056	62	16.50	2209	47	1	1	512	1697
25	65	2080	3969	4481	1120	64	17.34	2401	49	1	1	512	1889
26	67	2144	4233	4745	1186	66	18.14	2601	51	1	1	512	2089
27	69	2208	4505	5017	1254	68	18.89	2809	53	1	1	512	2297
28	71	2272	4785	5297	1324	70	19.60	3025	55	1	1	512	2513
29	73	2336	5073	5585	1396	72	20.28	3249	57	1	1	512	2737
30	75	2400	5369	5881	1470	74	20.92	3481	59	1	1	512	2969
31	77	2464	5673	6185	1546	76	21.52	3721	61	1	1	512	3209
32	79	2528	5985	6497	1624	78	22.10	3969	63	1	1	512	3457
33	81	2592	6305	6817	1704	80	22.65	4225	65	1	1	512	3713
34	83	2656	6633	7145	1786	82	23.18	4489	67	1	1	512	3977
35	85	2720	6969	7481	1870	84	23.68	4761	69	1	1	512	4249
36	87	2784	7313	7825	1956	86	24.16	5041	71	1	1	512	4529
37	89	2848	7665	8177	2044	88	24.62	5329	73	1	1	512	4817
38	91	2912	8025	8537	2134	90	25.06	5625	75	1	1	512	5113
39	93	2976	8393	8905	2226	92	25.48	5929	77	1	1	512	5417
40	95	3040	8769	9281	2320	94	25.88	6241	79	1	1	512	5729

Table 17.1 Triples Sequences Step 289: TS1; n(ν)=2ν+17; m=17

ν	n(ν)	x = 17n	y = $(n^2 - 289)/2$	r = $(n^2 + 289)/2$	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	19	323	36	325	81		-38.64	2	1	1	1	289	-287
2	21	357	76	365	91	10	-32.98	8	2	1	1	289	-281
3	23	391	120	409	102	11	-27.94	18	3	1	1	289	-271
4	25	425	168	457	114	12	-23.43	32	4	1	1	289	-257
5	27	459	220	509	127	13	-19.39	50	5	1	1	289	-239
6	29	493	276	565	141	14	-15.76	72	6	1	1	289	-217
7	31	527	336	625	156	15	-12.48	98	7	1	1	289	-191
8	33	561	400	689	172	16	-9.51	128	8	1	1	289	-161
9	35	595	468	757	189	17	-6.81	162	9	1	1	289	-127
10	37	629	540	829	207	18	-4.35	200	10	1	1	289	-89
11	39	663	616	905	226	19	-2.10	242	11	1	1	289	-47
12	41	**697**	**696**	**985**	246	20	**-0.04**	288	12	1	1	289	-1
13	43	731	780	1069	267	21	1.86	338	13	1	1	289	49
14	45	765	868	1157	289	22	3.61	392	14	1	1	289	103
15	47	799	960	1249	312	23	5.23	450	15	1	1	289	161
16	49	833	1056	1345	336	24	6.73	512	16	1	1	289	223
17	51	867	1156	1445	361	25	8.13	578	17	289	17	1	289
18	53	901	1260	1549	387	26	9.43	648	18	1	1	289	359
19	55	935	1368	1657	414	27	10.65	722	19	1	1	289	433
20	57	969	1480	1769	442	28	11.79	800	20	1	1	289	511
21	59	1003	1596	1885	471	29	12.85	882	21	1	1	289	593
22	61	1037	1716	2005	501	30	13.86	968	22	1	1	289	679
23	63	1071	1840	2129	532	31	14.80	1058	23	1	1	289	769
24	65	1105	1968	2257	564	32	15.69	1152	24	1	1	289	863
25	67	1139	2100	2389	597	33	16.53	1250	25	1	1	289	961
26	69	1173	2236	2525	631	34	17.32	1352	26	1	1	289	1063
27	71	1207	2376	2665	666	35	18.07	1458	27	1	1	289	1169
28	73	1241	2520	2809	702	36	18.78	1568	28	1	1	289	1279
29	75	1275	2668	2957	739	37	19.46	1682	29	1	1	289	1393
30	77	1309	2820	3109	777	38	20.10	1800	30	1	1	289	1511
31	79	1343	2976	3265	816	39	20.71	1922	31	1	1	289	1633
32	81	1377	3136	3425	856	40	21.30	2048	32	1	1	289	1759
33	83	1411	3300	3589	897	41	21.85	2178	33	1	1	289	1889
34	85	1445	3468	3757	939	42	22.38	2312	34	289	17	1	2023
35	87	1479	3640	3929	982	43	22.89	2450	35	1	1	289	2161
36	89	1513	3816	4105	1026	44	23.37	2592	36	1	1	289	2303
37	91	1547	3996	4285	1071	45	23.84	2738	37	1	1	289	2449
38	93	1581	4180	4469	1117	46	24.28	2888	38	1	1	289	2599
39	95	1615	4368	4657	1164	47	24.71	3042	39	1	1	289	2753
40	97	1649	4560	4849	1212	48	25.12	3200	40	1	1	289	2911

Table 17.2 Triples Sequences Step 578: TS3; n(ν)=2ν+16; m=17

ν	n(ν)	x = 34n	y = n^2 - 289	r = n^2 + 289	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	18	612	35	613	153		-41.73	1	1	1	1	578	-577
2	20	680	111	689	172	19	-35.73	9	3	1	1	578	-569
3	22	748	195	773	193	21	-30.39	25	5	1	1	578	-553
4	24	816	287	865	216	23	-25.62	49	7	1	1	578	-529
5	26	884	387	965	241	25	-21.36	81	9	1	1	578	-497
6	28	952	495	1073	268	27	-17.53	121	11	1	1	578	-457
7	30	1020	611	1189	297	29	-14.08	169	13	1	1	578	-409
8	32	1088	735	1313	328	31	-10.96	225	15	1	1	578	-353
9	34	1156	867	1445	361	33	-8.13	289	17	289	17	2	-289
10	36	1224	1007	1585	396	35	-5.55	361	19	1	1	578	-217
11	38	1292	1155	1733	433	37	-3.20	441	21	1	1	578	-137
12	40	1360	1311	1889	472	39	-1.05	529	23	1	1	578	-49
13	42	**1428**	**1475**	**2053**	513	41	**0.93**	625	25	1	1	578	47
14	44	1496	1647	2225	556	43	2.75	729	27	1	1	578	151
15	46	1564	1827	2405	601	45	4.44	841	29	1	1	578	263
16	48	1632	2015	2593	648	47	6.00	961	31	1	1	578	383
17	50	1700	2211	2789	697	49	7.45	1089	33	1	1	578	511
18	52	1768	2415	2993	748	51	8.79	1225	35	1	1	578	647
19	54	1836	2627	3205	801	53	10.05	1369	37	1	1	578	791
20	56	1904	2847	3425	856	55	11.23	1521	39	1	1	578	943
21	58	1972	3075	3653	913	57	12.33	1681	41	1	1	578	1103
22	60	2040	3311	3889	972	59	13.36	1849	43	1	1	578	1271
23	62	2108	3555	4133	1033	61	14.34	2025	45	1	1	578	1447
24	64	2176	3807	4385	1096	63	15.25	2209	47	1	1	578	1631
25	66	2244	4067	4645	1161	65	16.11	2401	49	1	1	578	1823
26	68	2312	4335	4913	1228	67	16.93	2601	51	289	17	2	2023
27	70	2380	4611	5189	1297	69	17.70	2809	53	1	1	578	2231
28	72	2448	4895	5473	1368	71	18.43	3025	55	1	1	578	2447
29	74	2516	5187	5765	1441	73	19.13	3249	57	1	1	578	2671
30	76	2584	5487	6065	1516	75	19.78	3481	59	1	1	578	2903
31	78	2652	5795	6373	1593	77	20.41	3721	61	1	1	578	3143
32	80	2720	6111	6689	1672	79	21.01	3969	63	1	1	578	3391
33	82	2788	6435	7013	1753	81	21.58	4225	65	1	1	578	3647
34	84	2856	6767	7345	1836	83	22.12	4489	67	1	1	578	3911
35	86	2924	7107	7685	1921	85	22.64	4761	69	1	1	578	4183
36	88	2992	7455	8033	2008	87	23.13	5041	71	1	1	578	4463
37	90	3060	7811	8389	2097	89	23.61	5329	73	1	1	578	4751
38	92	3128	8175	8753	2188	91	24.06	5625	75	1	1	578	5047
39	94	3196	8547	9125	2281	93	24.50	5929	77	1	1	578	5351
40	96	3264	8927	9505	2376	95	24.92	6241	79	1	1	578	5663

Table 18 Triples Sequences Step 648: TS2; $n(\nu) = 2\nu+17$; $m=2\cdot3\cdot3$

ν	$n(\nu)$	$x = 36n$	$y = n^2 - 324$	$r = n^2 + 324$	$k = (r - 1)/4$	Δk	γ	$s_s = r - x$	m_s	α	β	$s_p = (r - y)/\alpha$	y - x
1	19	684	37	685	171		-41.90	1	1	1	1	648	-647
2	21	756	117	765	191	20	-36.20	9	3	9	3	72	-639
3	23	828	205	853	213	22	-31.09	25	5	1	1	648	-623
4	25	900	301	949	237	24	-26.51	49	7	1	1	648	-599
5	27	972	405	1053	263	26	-22.38	81	9	81	9	8	-567
6	29	1044	517	1165	291	28	-18.65	121	11	1	1	648	-527
7	31	1116	637	1285	321	30	-15.28	169	13	1	1	648	-479
8	33	1188	765	1413	353	32	-12.22	225	15	9	3	72	-423
9	35	1260	901	1549	387	34	-9.43	289	17	1	1	648	-359
10	37	1332	1045	1693	423	36	-6.88	361	19	1	1	648	-287
11	39	1404	1197	1845	461	38	-4.55	441	21	9	3	72	-207
12	41	1476	1357	2005	501	40	-2.40	529	23	1	1	648	-119
13	43	**1548**	**1525**	**2173**	543	42	**-0.43**	625	25	1	1	648	-23
14	45	1620	1701	2349	587	44	1.40	729	27	81	9	8	81
15	47	1692	1885	2533	633	46	3.09	841	29	1	1	648	193
16	49	1764	2077	2725	681	48	4.66	961	31	1	1	648	313
17	51	1836	2277	2925	731	50	6.12	1089	33	9	3	72	441
18	53	1908	2485	3133	783	52	7.48	1225	35	1	1	648	577
19	55	1980	2701	3349	837	54	8.76	1369	37	1	1	648	721
20	57	2052	2925	3573	893	56	9.95	1521	39	9	3	72	873
21	59	2124	3157	3805	951	58	11.07	1681	41	1	1	648	1033
22	61	2196	3397	4045	1011	60	12.12	1849	43	1	1	648	1201
23	63	2268	3645	4293	1073	62	13.11	2025	45	81	9	8	1377
24	65	2340	3901	4549	1137	64	14.04	2209	47	1	1	648	1561
25	67	2412	4165	4813	1203	66	14.93	2401	49	1	1	648	1753
26	69	2484	4437	5085	1271	68	15.76	2601	51	9	3	72	1953
27	71	2556	4717	5365	1341	70	16.55	2809	53	1	1	648	2161
28	73	2628	5005	5653	1413	72	17.30	3025	55	1	1	648	2377
29	75	2700	5301	5949	1487	74	18.01	3249	57	9	3	72	2601
30	77	2772	5605	6253	1563	76	18.69	3481	59	1	1	648	2833
31	79	2844	5917	6565	1641	78	19.33	3721	61	1	1	648	3073
32	81	2916	6237	6885	1721	80	19.94	3969	63	81	9	8	3321
33	83	2988	6565	7213	1803	82	20.53	4225	65	1	1	648	3577
34	85	3060	6901	7549	1887	84	21.09	4489	67	1	1	648	3841
35	87	3132	7245	7893	1973	86	21.62	4761	69	9	3	72	4113
36	89	3204	7597	8245	2061	88	22.13	5041	71	1	1	648	4393
37	91	3276	7957	8605	2151	90	22.62	5329	73	1	1	648	4681
38	93	3348	8325	8973	2243	92	23.09	5625	75	9	3	72	4977
39	95	3420	8701	9349	2337	94	23.54	5929	77	1	1	648	5281
40	97	3492	9085	9733	2433	96	23.98	6241	79	1	1	648	5593

Table 19.1 Triples Sequences Step 361: TS1; $n(\nu) = 2\nu+19$; m=19

ν	$n(\nu)$	x = 19n	y = $(n^2- 361)/2$	r = $(n^2+ 361)/2$	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	$s_p = (r - y)/\alpha$	y - x
1	21	399	40	401	100		-39.28	2	1	1	1	361	-359
2	23	437	84	445	111	11	-34.12	8	2	1	1	361	-353
3	25	475	132	493	123	12	-29.47	18	3	1	1	361	-343
4	27	513	184	545	136	13	-25.27	32	4	1	1	361	-329
5	29	551	240	601	150	14	-21.46	50	5	1	1	361	-311
6	31	589	300	661	165	15	-18.01	72	6	1	1	361	-289
7	33	627	364	725	181	16	-14.86	98	7	1	1	361	-263
8	35	665	432	793	198	17	-11.99	128	8	1	1	361	-233
9	37	703	504	865	216	18	-9.36	162	9	1	1	361	-199
10	39	741	580	941	235	19	-6.95	200	10	1	1	361	-161
11	41	779	660	1021	255	20	-4.73	242	11	1	1	361	-119
12	43	817	744	1105	276	21	-2.68	288	12	1	1	361	-73
13	45	**855**	**832**	**1193**	298	22	**-0.78**	338	13	1	1	361	-23
14	47	893	924	1285	321	23	0.98	392	14	1	1	361	31
15	49	931	1020	1381	345	24	2.61	450	15	1	1	361	89
16	51	969	1120	1481	370	25	4.14	512	16	1	1	361	151
17	53	1007	1224	1585	396	26	5.56	578	17	1	1	361	217
18	55	1045	1332	1693	423	27	6.89	648	18	1	1	361	287
19	57	1083	1444	1805	451	28	8.13	722	19	361	19	1	361
20	59	1121	1560	1921	480	29	9.30	800	20	1	1	361	439
21	61	1159	1680	2041	510	30	10.40	882	21	1	1	361	521
22	63	1197	1804	2165	541	31	11.44	968	22	1	1	361	607
23	65	1235	1932	2293	573	32	12.41	1058	23	1	1	361	697
24	67	1273	2064	2425	606	33	13.34	1152	24	1	1	361	791
25	69	1311	2200	2561	640	34	14.21	1250	25	1	1	361	889
26	71	1349	2340	2701	675	35	15.04	1352	26	1	1	361	991
27	73	1387	2484	2845	711	36	15.82	1458	27	1	1	361	1097
28	75	1425	2632	2993	748	37	16.57	1568	28	1	1	361	1207
29	77	1463	2784	3145	786	38	17.28	1682	29	1	1	361	1321
30	79	1501	2940	3301	825	39	17.96	1800	30	1	1	361	1439
31	81	1539	3100	3461	865	40	18.60	1922	31	1	1	361	1561
32	83	1577	3264	3625	906	41	19.21	2048	32	1	1	361	1687
33	85	1615	3432	3793	948	42	19.80	2178	33	1	1	361	1817
34	87	1653	3604	3965	991	43	20.36	2312	34	1	1	361	1951
35	89	1691	3780	4141	1035	44	20.90	2450	35	1	1	361	2089
36	91	1729	3960	4321	1080	45	21.42	2592	36	1	1	361	2231
37	93	1767	4144	4505	1126	46	21.91	2738	37	1	1	361	2377
38	95	1805	4332	4693	1173	47	22.38	2888	38	361	19	1	2527
39	97	1843	4524	4885	1221	48	22.84	3042	39	1	1	361	2681
40	99	1881	4720	5081	1270	49	23.27	3200	40	1	1	361	2839

Table 19.2 Triples Sequences Step 722: TS3; n(ν) = 2ν+18; m=19

ν	n(ν)	x = 38n	y = n^2 - 361	r = n^2 + 361	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	20	760	39	761	190		-42.06	1	1	1	1	722	-721
2	22	836	123	845	211	21	-36.63	9	3	1	1	722	-713
3	24	912	215	937	234	23	-31.73	25	5	1	1	722	-697
4	26	988	315	1037	259	25	-27.32	49	7	1	1	722	-673
5	28	1064	423	1145	286	27	-23.32	81	9	1	1	722	-641
6	30	1140	539	1261	315	29	-19.69	121	11	1	1	722	-601
7	32	1216	663	1385	346	31	-16.40	169	13	1	1	722	-553
8	34	1292	795	1517	379	33	-13.39	225	15	1	1	722	-497
9	36	1368	935	1657	414	35	-10.65	289	17	1	1	722	-433
10	38	1444	1083	1805	451	37	-8.13	361	19	361	19	2	-361
11	40	1520	1239	1961	490	39	-5.81	441	21	1	1	722	-281
12	42	1596	1403	2125	531	41	-3.68	529	23	1	1	722	-193
13	44	1672	1575	2297	574	43	-1.71	625	25	1	1	722	-97
14	46	**1748**	**1755**	**2477**	619	45	**0.12**	729	27	1	1	722	7
15	48	1824	1943	2665	666	47	1.81	841	29	1	1	722	119
16	50	1900	2139	2861	715	49	3.39	961	31	1	1	722	239
17	52	1976	2343	3065	766	51	4.86	1089	33	1	1	722	367
18	54	2052	2555	3277	819	53	6.23	1225	35	1	1	722	503
19	56	2128	2775	3497	874	55	7.52	1369	37	1	1	722	647
20	58	2204	3003	3725	931	57	8.73	1521	39	1	1	722	799
21	60	2280	3239	3961	990	59	9.86	1681	41	1	1	722	959
22	62	2356	3483	4205	1051	61	10.93	1849	43	1	1	722	1127
23	64	2432	3735	4457	1114	63	11.93	2025	45	1	1	722	1303
24	66	2508	3995	4717	1179	65	12.88	2209	47	1	1	722	1487
25	68	2584	4263	4985	1246	67	13.78	2401	49	1	1	722	1679
26	70	2660	4539	5261	1315	69	14.63	2601	51	1	1	722	1879
27	72	2736	4823	5545	1386	71	15.44	2809	53	1	1	722	2087
28	74	2812	5115	5837	1459	73	16.20	3025	55	1	1	722	2303
29	76	2888	5415	6137	1534	75	16.93	3249	57	361	19	2	2527
30	78	2964	5723	6445	1611	77	17.62	3481	59	1	1	722	2759
31	80	3040	6039	6761	1690	79	18.28	3721	61	1	1	722	2999
32	82	3116	6363	7085	1771	81	18.91	3969	63	1	1	722	3247
33	84	3192	6695	7417	1854	83	19.51	4225	65	1	1	722	3503
34	86	3268	7035	7757	1939	85	20.09	4489	67	1	1	722	3767
35	88	3344	7383	8105	2026	87	20.63	4761	69	1	1	722	4039
36	90	3420	7739	8461	2115	89	21.16	5041	71	1	1	722	4319
37	92	3496	8103	8825	2206	91	21.66	5329	73	1	1	722	4607
38	94	3572	8475	9197	2299	93	22.15	5625	75	1	1	722	4903
39	96	3648	8855	9577	2394	95	22.61	5929	77	1	1	722	5207
40	98	3724	9243	9965	2491	97	23.06	6241	79	1	1	722	5519

Table 20 Triples Sequences Step 800: TS2; n(ν) = 2ν+19; m=2·2·5

ν	n(ν)	x = 40n	y = n^2- 400	r = n^2+ 400	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	21	840	41	841	210		-42.21	1	1	1	1	800	-799
2	23	920	129	929	232	22	-37.02	9	3	1	1	800	-791
3	25	1000	225	1025	256	24	-32.32	25	5	25	5	32	-775
4	27	1080	329	1129	282	26	-28.06	49	7	1	1	800	-751
5	29	1160	441	1241	310	28	-24.18	81	9	1	1	800	-719
6	31	1240	561	1361	340	30	-20.66	121	11	1	1	800	-679
7	33	1320	689	1489	372	32	-17.44	169	13	1	1	800	-631
8	35	1400	825	1625	406	34	-14.49	225	15	25	5	32	-575
9	37	1480	969	1769	442	36	-11.79	289	17	1	1	800	-511
10	39	1560	1121	1921	480	38	-9.30	361	19	1	1	800	-439
11	41	1640	1281	2081	520	40	-7.01	441	21	1	1	800	-359
12	43	1720	1449	2249	562	42	-4.89	529	23	1	1	800	-271
13	45	1800	1625	2425	606	44	-2.92	625	25	25	5	32	-175
14	47	1880	1809	2609	652	46	-1.10	729	27	1	1	800	-71
15	49	**1960**	**2001**	**2801**	700	48	**0.59**	841	29	1	1	800	41
16	51	2040	2201	3001	750	50	2.18	961	31	1	1	800	161
17	53	2120	2409	3209	802	52	3.65	1089	33	1	1	800	289
18	55	2200	2625	3425	856	54	5.04	1225	35	25	5	32	425
19	57	2280	2849	3649	912	56	6.33	1369	37	1	1	800	569
20	59	2360	3081	3881	970	58	7.55	1521	39	1	1	800	721
21	61	2440	3321	4121	1030	60	8.70	1681	41	1	1	800	881
22	63	2520	3569	4369	1092	62	9.78	1849	43	1	1	800	1049
23	65	2600	3825	4625	1156	64	10.80	2025	45	25	5	32	1225
24	67	2680	4089	4889	1222	66	11.76	2209	47	1	1	800	1409
25	69	2760	4361	5161	1290	68	12.67	2401	49	1	1	800	1601
26	71	2840	4641	5441	1360	70	13.54	2601	51	1	1	800	1801
27	73	2920	4929	5729	1432	72	14.36	2809	53	1	1	800	2009
28	75	3000	5225	6025	1506	74	15.14	3025	55	25	5	32	2225
29	77	3080	5529	6329	1582	76	15.88	3249	57	1	1	800	2449
30	79	3160	5841	6641	1660	78	16.59	3481	59	1	1	800	2681
31	81	3240	6161	6961	1740	80	17.26	3721	61	1	1	800	2921
32	83	3320	6489	7289	1822	82	17.91	3969	63	1	1	800	3169
33	85	3400	6825	7625	1906	84	18.52	4225	65	25	5	32	3425
34	87	3480	7169	7969	1992	86	19.11	4489	67	1	1	800	3689
35	89	3560	7521	8321	2080	88	19.67	4761	69	1	1	800	3961
36	91	3640	7881	8681	2170	90	20.21	5041	71	1	1	800	4241
37	93	3720	8249	9049	2262	92	20.73	5329	73	1	1	800	4529
38	95	3800	8625	9425	2356	94	21.22	5625	75	25	5	32	4825
39	97	3880	9009	9809	2452	96	21.70	5929	77	1	1	800	5129
40	99	3960	9401	10201	2550	98	22.16	6241	79	1	1	800	5441

Table 21.1 Triples Sequences Step 441: TS1; $n(\nu)=2\nu+21$; $m=3\cdot7$

ν	$n(\nu)$	$x = 21n$	$y = (n^2 - 441)/2$	$r = (n^2 + 441)/2$	$k = (r - 1)/4$	Δk	γ	$s_s = r - x$	m_s	α	β	$s_p = (r - y)/\alpha$	y - x
1	23	483	44	485	121		-39.79	2	1	1	1	441	-439
2	25	525	92	533	133	12	-35.06	8	2	1	1	441	-433
3	27	567	144	585	146	13	-30.75	18	3	9	3	49	-423
4	29	609	200	641	160	14	-26.82	32	4	1	1	441	-409
5	31	651	260	701	175	15	-23.23	50	5	1	1	441	-391
6	33	693	324	765	191	16	-19.94	72	6	9	3	49	-369
7	35	735	392	833	208	17	-16.93	98	7	49	7	9	-343
8	37	777	464	905	226	18	-14.15	128	8	1	1	441	-313
9	39	819	540	981	245	19	-11.60	162	9	9	3	49	-279
10	41	861	620	1061	265	20	-9.24	200	10	1	1	441	-241
11	43	903	704	1145	286	21	-7.06	242	11	1	1	441	-199
12	45	945	792	1233	308	22	-5.03	288	12	9	3	49	-153
13	47	987	884	1325	331	23	-3.15	338	13	1	1	441	-103
14	49	1029	980	1421	355	24	-1.40	392	14	49	7	9	-49
15	51	1071	1080	1521	380	25	0.24	450	15	9	3	49	9
16	53	**1113**	**1184**	**1625**	406	26	**1.77**	512	16	1	1	441	71
17	55	1155	1292	1733	433	27	3.21	578	17	1	1	441	137
18	57	1197	1404	1845	461	28	4.55	648	18	9	3	49	207
19	59	1239	1520	1961	490	29	5.82	722	19	1	1	441	281
20	61	1281	1640	2081	520	30	7.01	800	20	1	1	441	359
21	63	1323	1764	2205	551	31	8.13	882	21	441	21	1	441
22	65	1365	1892	2333	583	32	9.19	968	22	1	1	441	527
23	67	1407	2024	2465	616	33	10.20	1058	23	1	1	441	617
24	69	1449	2160	2601	650	34	11.15	1152	24	9	3	49	711
25	71	1491	2300	2741	685	35	12.05	1250	25	1	1	441	809
26	73	1533	2444	2885	721	36	12.90	1352	26	1	1	441	911
27	75	1575	2592	3033	758	37	13.72	1458	27	9	3	49	1017
28	77	1617	2744	3185	796	38	14.49	1568	28	49	7	9	1127
29	79	1659	2900	3341	835	39	15.23	1682	29	1	1	441	1241
30	81	1701	3060	3501	875	40	15.93	1800	30	9	3	49	1359
31	83	1743	3224	3665	916	41	16.60	1922	31	1	1	441	1481
32	85	1785	3392	3833	958	42	17.25	2048	32	1	1	441	1607
33	87	1827	3564	4005	1001	43	17.86	2178	33	9	3	49	1737
34	89	1869	3740	4181	1045	44	18.45	2312	34	1	1	441	1871
35	91	1911	3920	4361	1090	45	19.01	2450	35	49	7	9	2009
36	93	1953	4104	4545	1136	46	19.55	2592	36	9	3	49	2151
37	95	1995	4292	4733	1183	47	20.07	2738	37	1	1	441	2297
38	97	2037	4484	4925	1231	48	20.57	2888	38	1	1	441	2447
39	99	2079	4680	5121	1280	49	21.05	3042	39	9	3	49	2601
40	101	2121	4880	5321	1330	50	21.51	3200	40	1	1	441	2759

Table 21.2 Triples Sequences Step 882: TS3; $n(\nu)=2\nu+20$; $m=3\cdot7$

ν	$n(\nu)$	$x = 42n$	$y = n^2 - 441$	$r = n^2 + 441$	$k = (r - 1)/4$	Δk	γ	$s_S = r - x$	m_S	α	β	$s_P = (r - y)/\alpha$	$y - x$
1	22	924	43	925	231		-42.34	1	1	1	1	882	-881
2	24	1008	135	1017	254	23	-37.37	9	3	9	3	98	-873
3	26	1092	235	1117	279	25	-32.85	25	5	1	1	882	-857
4	28	1176	343	1225	306	27	-28.74	49	7	49	7	18	-833
5	30	1260	459	1341	335	29	-24.98	81	9	9	3	98	-801
6	32	1344	583	1465	366	31	-21.55	121	11	1	1	882	-761
7	34	1428	715	1597	399	33	-18.40	169	13	1	1	882	-713
8	36	1512	855	1737	434	35	-15.51	225	15	9	3	98	-657
9	38	1596	1003	1885	471	37	-12.85	289	17	1	1	882	-593
10	40	1680	1159	2041	510	39	-10.40	361	19	1	1	882	-521
11	42	1764	1323	2205	551	41	-8.13	441	21	441	21	2	-441
12	44	1848	1495	2377	594	43	-6.03	529	23	1	1	882	-353
13	46	1932	1675	2557	639	45	-4.07	625	25	1	1	882	-257
14	48	2016	1863	2745	686	47	-2.26	729	27	9	3	98	-153
15	50	**2100**	**2059**	**2941**	735	49	**-0.56**	841	29	1	1	882	-41
16	52	2184	2263	3145	786	51	1.02	961	31	1	1	882	79
17	54	2268	2475	3357	839	53	2.50	1089	33	9	3	98	207
18	56	2352	2695	3577	894	55	3.89	1225	35	49	7	18	343
19	58	2436	2923	3805	951	57	5.19	1369	37	1	1	882	487
20	60	2520	3159	4041	1010	59	6.42	1521	39	9	3	98	639
21	62	2604	3403	4285	1071	61	7.58	1681	41	1	1	882	799
22	64	2688	3655	4537	1134	63	8.67	1849	43	1	1	882	967
23	66	2772	3915	4797	1199	65	9.70	2025	45	9	3	98	1143
24	68	2856	4183	5065	1266	67	10.68	2209	47	1	1	882	1327
25	70	2940	4459	5341	1335	69	11.60	2401	49	49	7	18	1519
26	72	3024	4743	5625	1406	71	12.48	2601	51	9	3	98	1719
27	74	3108	5035	5917	1479	73	13.32	2809	53	1	1	882	1927
28	76	3192	5335	6217	1554	75	14.11	3025	55	1	1	882	2143
29	78	3276	5643	6525	1631	77	14.86	3249	57	9	3	98	2367
30	80	3360	5959	6841	1710	79	15.59	3481	59	1	1	882	2599
31	82	3444	6283	7165	1791	81	16.27	3721	61	1	1	882	2839
32	84	3528	6615	7497	1874	83	16.93	3969	63	441	21	2	3087
33	86	3612	6955	7837	1959	85	17.56	4225	65	1	1	882	3343
34	88	3696	7303	8185	2046	87	18.16	4489	67	1	1	882	3607
35	90	3780	7659	8541	2135	89	18.73	4761	69	9	3	98	3879
36	92	3864	8023	8905	2226	91	19.29	5041	71	1	1	882	4159
37	94	3948	8395	9277	2319	93	19.82	5329	73	1	1	882	4447
38	96	4032	8775	9657	2414	95	20.32	5625	75	9	3	98	4743
39	98	4116	9163	10045	2511	97	20.81	5929	77	49	7	18	5047
40	100	4200	9559	10441	2610	99	21.28	6241	79	1	1	882	5359

Table 22 Triples Sequences Step 968: TS2; n(ν)=2ν+21; m=2·11

ν	n(ν)	x = 44n	y = n^2- 484	r = n^2+ 484	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	23	1012	45	1013	253		-42.45	1	1	1	1	968	-967
2	25	1100	141	1109	277	24	-37.70	9	3	1	1	968	-959
3	27	1188	245	1213	303	26	-33.35	25	5	1	1	968	-943
4	29	1276	357	1325	331	28	-29.37	49	7	1	1	968	-919
5	31	1364	477	1445	361	30	-25.72	81	9	1	1	968	-887
6	33	1452	605	1573	393	32	-22.38	121	11	121	11	8	-847
7	35	1540	741	1709	427	34	-19.30	169	13	1	1	968	-799
8	37	1628	885	1853	463	36	-16.47	225	15	1	1	968	-743
9	39	1716	1037	2005	501	38	-13.85	289	17	1	1	968	-679
10	41	1804	1197	2165	541	40	-11.43	361	19	1	1	968	-607
11	43	1892	1365	2333	583	42	-9.19	441	21	1	1	968	-527
12	45	1980	1541	2509	627	44	-7.11	529	23	1	1	968	-439
13	47	2068	1725	2693	673	46	-5.17	625	25	1	1	968	-343
14	49	2156	1917	2885	721	48	-3.36	729	27	1	1	968	-239
15	51	2244	2117	3085	771	50	-1.67	841	29	1	1	968	-127
16	53	**2332**	**2325**	**3293**	823	52	**-0.08**	961	31	1	1	968	-7
17	55	2420	2541	3509	877	54	1.40	1089	33	121	11	8	121
18	57	2508	2765	3733	933	56	2.79	1225	35	1	1	968	257
19	59	2596	2997	3965	991	58	4.10	1369	37	1	1	968	401
20	61	2684	3237	4205	1051	60	5.34	1521	39	1	1	968	553
21	63	2772	3485	4453	1113	62	6.50	1681	41	1	1	968	713
22	65	2860	3741	4709	1177	64	7.60	1849	43	1	1	968	881
23	67	2948	4005	4973	1243	66	8.65	2025	45	1	1	968	1057
24	69	3036	4277	5245	1311	68	9.63	2209	47	1	1	968	1241
25	71	3124	4557	5525	1381	70	10.57	2401	49	1	1	968	1433
26	73	3212	4845	5813	1453	72	11.46	2601	51	1	1	968	1633
27	75	3300	5141	6109	1527	74	12.31	2809	53	1	1	968	1841
28	77	3388	5445	6413	1603	76	13.11	3025	55	121	11	8	2057
29	79	3476	5757	6725	1681	78	13.88	3249	57	1	1	968	2281
30	81	3564	6077	7045	1761	80	14.61	3481	59	1	1	968	2513
31	83	3652	6405	7373	1843	82	15.31	3721	61	1	1	968	2753
32	85	3740	6741	7709	1927	84	15.98	3969	63	1	1	968	3001
33	87	3828	7085	8053	2013	86	16.62	4225	65	1	1	968	3257
34	89	3916	7437	8405	2101	88	17.23	4489	67	1	1	968	3521
35	91	4004	7797	8765	2191	90	17.82	4761	69	1	1	968	3793
36	93	4092	8165	9133	2283	92	18.38	5041	71	1	1	968	4073
37	95	4180	8541	9509	2377	94	18.92	5329	73	1	1	968	4361
38	97	4268	8925	9893	2473	96	19.44	5625	75	1	1	968	4657
39	99	4356	9317	10285	2571	98	19.94	5929	77	121	11	8	4961
40	101	4444	9717	10685	2671	100	20.43	6241	79	1	1	968	5273

Table 23.1 Triples Sequences Step 529: TS1; n(ν)=2ν+23; m=23

ν	n(ν)	x = 23n	y = $(n^2 - 529)/2$	r = $(n^2 + 529)/2$	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	25	575	48	577	144		-40.23	2	1	1	1	529	-527
2	27	621	100	629	157	13	-35.85	8	2	1	1	529	-521
3	29	667	156	685	171	14	-31.84	18	3	1	1	529	-511
4	31	713	216	745	186	15	-28.15	32	4	1	1	529	-497
5	33	759	280	809	202	16	-24.75	50	5	1	1	529	-479
6	35	805	348	877	219	17	-21.62	72	6	1	1	529	-457
7	37	851	420	949	237	18	-18.73	98	7	1	1	529	-431
8	39	897	496	1025	256	19	-16.06	128	8	1	1	529	-401
9	41	943	576	1105	276	20	-13.58	162	9	1	1	529	-367
10	43	989	660	1189	297	21	-11.28	200	10	1	1	529	-329
11	45	1035	748	1277	319	22	-9.14	242	11	1	1	529	-287
12	47	1081	840	1369	342	23	-7.15	288	12	1	1	529	-241
13	49	1127	936	1465	366	24	-5.29	338	13	1	1	529	-191
14	51	1173	1036	1565	391	25	-3.55	392	14	1	1	529	-137
15	53	1219	1140	1669	417	26	-1.92	450	15	1	1	529	-79
16	55	**1265**	**1248**	**1777**	444	27	**-0.39**	512	16	1	1	529	-17
17	57	1311	1360	1889	472	28	1.05	578	17	1	1	529	49
18	59	1357	1476	2005	501	29	2.41	648	18	1	1	529	119
19	61	1403	1596	2125	531	30	3.68	722	19	1	1	529	193
20	63	1449	1720	2249	562	31	4.89	800	20	1	1	529	271
21	65	1495	1848	2377	594	32	6.03	882	21	1	1	529	353
22	67	1541	1980	2509	627	33	7.11	968	22	1	1	529	439
23	69	1587	2116	2645	661	34	8.13	1058	23	529	23	1	529
24	71	1633	2256	2785	696	35	9.10	1152	24	1	1	529	623
25	73	1679	2400	2929	732	36	10.03	1250	25	1	1	529	721
26	75	1725	2548	3077	769	37	10.90	1352	26	1	1	529	823
27	77	1771	2700	3229	807	38	11.74	1458	27	1	1	529	929
28	79	1817	2856	3385	846	39	12.54	1568	28	1	1	529	1039
29	81	1863	3016	3545	886	40	13.30	1682	29	1	1	529	1153
30	83	1909	3180	3709	927	41	14.02	1800	30	1	1	529	1271
31	85	1955	3348	3877	969	42	14.72	1922	31	1	1	529	1393
32	87	2001	3520	4049	1012	43	15.39	2048	32	1	1	529	1519
33	89	2047	3696	4225	1056	44	16.02	2178	33	1	1	529	1649
34	91	2093	3876	4405	1101	45	16.63	2312	34	1	1	529	1783
35	93	2139	4060	4589	1147	46	17.22	2450	35	1	1	529	1921
36	95	2185	4248	4777	1194	47	17.78	2592	36	1	1	529	2063
37	97	2231	4440	4969	1242	48	18.32	2738	37	1	1	529	2209
38	99	2277	4636	5165	1291	49	18.84	2888	38	1	1	529	2359
39	101	2323	4836	5365	1341	50	19.34	3042	39	1	1	529	2513
40	103	2369	5040	5569	1392	51	19.83	3200	40	1	1	529	2671

Table 23.2 Triples Sequences Step 1058: TS3; n(ν)=2ν+22; m=23

ν	n(ν)	x = 46n	y = n^2- 529	r = n^2+ 529	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	24	1104	47	1105	276		-42.56	1	1	1	1	1058	-1057
2	26	1196	147	1205	301	25	-37.99	9	3	1	1	1058	-1049
3	28	1288	255	1313	328	27	-33.80	25	5	1	1	1058	-1033
4	30	1380	371	1429	357	29	-29.95	49	7	1	1	1058	-1009
5	32	1472	495	1553	388	31	-26.41	81	9	1	1	1058	-977
6	34	1564	627	1685	421	33	-23.15	121	11	1	1	1058	-937
7	36	1656	767	1825	456	35	-20.15	169	13	1	1	1058	-889
8	38	1748	915	1973	493	37	-17.37	225	15	1	1	1058	-833
9	40	1840	1071	2129	532	39	-14.80	289	17	1	1	1058	-769
10	42	1932	1235	2293	573	41	-12.41	361	19	1	1	1058	-697
11	44	2024	1407	2465	616	43	-10.19	441	21	1	1	1058	-617
12	46	2116	1587	2645	661	45	-8.13	529	23	529	23	2	-529
13	48	2208	1775	2833	708	47	-6.20	625	25	1	1	1058	-433
14	50	2300	1971	3029	757	49	-4.40	729	27	1	1	1058	-329
15	52	2392	2175	3233	808	51	-2.72	841	29	1	1	1058	-217
16	54	2484	2387	3445	861	53	-1.14	961	31	1	1	1058	-97
17	**56**	**2576**	**2607**	**3665**	916	55	**0.34**	1089	33	**1**	**1**	1058	31
18	58	2668	2835	3893	973	57	1.74	1225	35	1	1	1058	167
19	60	2760	3071	4129	1032	59	3.05	1369	37	1	1	1058	311
20	62	2852	3315	4373	1093	61	4.29	1521	39	1	1	1058	463
21	64	2944	3567	4625	1156	63	5.47	1681	41	1	1	1058	623
22	66	3036	3827	4885	1221	65	6.58	1849	43	1	1	1058	791
23	68	3128	4095	5153	1288	67	7.63	2025	45	1	1	1058	967
24	70	3220	4371	5429	1357	69	8.62	2209	47	1	1	1058	1151
25	72	3312	4655	5713	1428	71	9.57	2401	49	1	1	1058	1343
26	74	3404	4947	6005	1501	73	10.47	2601	51	1	1	1058	1543
27	76	3496	5247	6305	1576	75	11.33	2809	53	1	1	1058	1751
28	78	3588	5555	6613	1653	77	12.14	3025	55	1	1	1058	1967
29	80	3680	5871	6929	1732	79	12.92	3249	57	1	1	1058	2191
30	82	3772	6195	7253	1813	81	13.67	3481	59	1	1	1058	2423
31	84	3864	6527	7585	1896	83	14.38	3721	61	1	1	1058	2663
32	86	3956	6867	7925	1981	85	15.06	3969	63	1	1	1058	2911
33	88	4048	7215	8273	2068	87	15.71	4225	65	1	1	1058	3167
34	90	4140	7571	8629	2157	89	16.33	4489	67	1	1	1058	3431
35	92	4232	7935	8993	2248	91	16.93	4761	69	529	23	2	3703
36	94	4324	8307	9365	2341	93	17.50	5041	71	1	1	1058	3983
37	96	4416	8687	9745	2436	95	18.06	5329	73	1	1	1058	4271
38	98	4508	9075	10133	2533	97	18.59	5625	75	1	1	1058	4567
39	100	4600	9471	10529	2632	99	19.10	5929	77	1	1	1058	4871
40	102	4692	9875	10933	2733	101	19.59	6241	79	1	1	1058	5183

Table 24 Triples Sequences Step 1152: TS2; n(ν)=2ν+23; m=2·2·2·3

ν	n(ν)	x = 48n	y = n^2- 576	r = n^2+ 576	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	25	1200	49	1201	300		-42.66	1	1	1	1	1152	-1151
2	27	1296	153	1305	326	26	-38.27	9	3	9	3	128	-1143
3	29	1392	265	1417	354	28	-34.22	25	5	1	1	1152	-1127
4	31	1488	385	1537	384	30	-30.49	49	7	1	1	1152	-1103
5	33	1584	513	1665	416	32	-27.05	81	9	9	3	128	-1071
6	35	1680	649	1801	450	34	-23.88	121	11	1	1	1152	-1031
7	37	1776	793	1945	486	36	-20.94	169	13	1	1	1152	-983
8	39	1872	945	2097	524	38	-18.21	225	15	9	3	128	-927
9	41	1968	1105	2257	564	40	-15.69	289	17	1	1	1152	-863
10	43	2064	1273	2425	606	42	-13.33	361	19	1	1	1152	-791
11	45	2160	1449	2601	650	44	-11.14	441	21	9	3	128	-711
12	47	2256	1633	2785	696	46	-9.10	529	23	1	1	1152	-623
13	49	2352	1825	2977	744	48	-7.19	625	25	1	1	1152	-527
14	51	2448	2025	3177	794	50	-5.40	729	27	9	3	128	-423
15	53	2544	2233	3385	846	52	-3.72	841	29	1	1	1152	-311
16	55	2640	2449	3601	900	54	-2.15	961	31	1	1	1152	-191
17	57	2736	2673	3825	956	56	-0.67	1089	33	9	3	128	-63
18	59	**2832**	**2905**	**4057**	1014	58	**0.73**	1225	35	1	1	1152	73
19	61	2928	3145	4297	1074	60	2.05	1369	37	1	1	1152	217
20	63	3024	3393	4545	1136	62	3.29	1521	39	9	3	128	369
21	65	3120	3649	4801	1200	64	4.47	1681	41	1	1	1152	529
22	67	3216	3913	5065	1266	66	5.59	1849	43	1	1	1152	697
23	69	3312	4185	5337	1334	68	6.64	2025	45	9	3	128	873
24	71	3408	4465	5617	1404	70	7.65	2209	47	1	1	1152	1057
25	73	3504	4753	5905	1476	72	8.60	2401	49	1	1	1152	1249
26	75	3600	5049	6201	1550	74	9.51	2601	51	9	3	128	1449
27	77	3696	5353	6505	1626	76	10.38	2809	53	1	1	1152	1657
28	79	3792	5665	6817	1704	78	11.20	3025	55	1	1	1152	1873
29	81	3888	5985	7137	1784	80	11.99	3249	57	9	3	128	2097
30	83	3984	6313	7465	1866	82	12.75	3481	59	1	1	1152	2329
31	85	4080	6649	7801	1950	84	13.47	3721	61	1	1	1152	2569
32	87	4176	6993	8145	2036	86	14.16	3969	63	9	3	128	2817
33	89	4272	7345	8497	2124	88	14.82	4225	65	1	1	1152	3073
34	91	4368	7705	8857	2214	90	15.45	4489	67	1	1	1152	3337
35	93	4464	8073	9225	2306	92	16.06	4761	69	9	3	128	3609
36	95	4560	8449	9601	2400	94	16.65	5041	71	1	1	1152	3889
37	97	4656	8833	9985	2496	96	17.21	5329	73	1	1	1152	4177
38	99	4752	9225	10377	2594	98	17.75	5625	75	9	3	128	4473
39	101	4848	9625	10777	2694	100	18.27	5929	77	1	1	1152	4777
40	103	4944	10033	11185	2796	102	18.77	6241	79	1	1	1152	5089

Table 25.1 Triples Sequences Step 625: TS1; n(ν)=2ν+25; m= 5·5

ν	n(ν)	x = 25n	y = (n^2- 625)/2	r = (n^2+ 625)/2	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	27	675	52	677	169		-40.59	2	1	1	1	625	-623
2	29	725	108	733	183	14	-36.53	8	2	1	1	625	-617
3	31	775	168	793	198	15	-32.77	18	3	1	1	625	-607
4	33	825	232	857	214	16	-29.29	32	4	1	1	625	-593
5	35	875	300	925	231	17	-26.07	50	5	25	5	25	-575
6	37	925	372	997	249	18	-23.09	72	6	1	1	625	-553
7	39	975	448	1073	268	19	-20.32	98	7	1	1	625	-527
8	41	1025	528	1153	288	20	-17.75	128	8	1	1	625	-497
9	43	1075	612	1237	309	21	-15.35	162	9	1	1	625	-463
10	45	1125	700	1325	331	22	-13.11	200	10	25	5	25	-425
11	47	1175	792	1417	354	23	-11.02	242	11	1	1	625	-383
12	49	1225	888	1513	378	24	-9.06	288	12	1	1	625	-337
13	51	1275	988	1613	403	25	-7.23	338	13	1	1	625	-287
14	53	1325	1092	1717	429	26	-5.51	392	14	1	1	625	-233
15	55	1375	1200	1825	456	27	-3.89	450	15	25	5	25	-175
16	57	1425	1312	1937	484	28	-2.36	512	16	1	1	625	-113
17	59	1475	1428	2053	513	29	-0.93	578	17	1	1	625	-47
18	61	**1525**	**1548**	**2173**	543	30	**0.43**	648	18	1	1	625	23
19	63	1575	1672	2297	574	31	1.71	722	19	1	1	625	97
20	65	1625	1800	2425	606	32	2.93	800	20	25	5	25	175
21	67	1675	1932	2557	639	33	4.08	882	21	1	1	625	257
22	69	1725	2068	2693	673	34	5.17	968	22	1	1	625	343
23	71	1775	2208	2833	708	35	6.21	1058	23	1	1	625	433
24	73	1825	2352	2977	744	36	7.19	1152	24	1	1	625	527
25	75	1875	2500	3125	781	37	8.13	1250	25	625	25	1	625
26	77	1925	2652	3277	819	38	9.03	1352	26	1	1	625	727
27	79	1975	2808	3433	858	39	9.88	1458	27	1	1	625	833
28	81	2025	2968	3593	898	40	10.70	1568	28	1	1	625	943
29	83	2075	3132	3757	939	41	11.48	1682	29	1	1	625	1057
30	85	2125	3300	3925	981	42	12.22	1800	30	25	5	25	1175
31	87	2175	3472	4097	1024	43	12.94	1922	31	1	1	625	1297
32	89	2225	3648	4273	1068	44	13.62	2048	32	1	1	625	1423
33	91	2275	3828	4453	1113	45	14.28	2178	33	1	1	625	1553
34	93	2325	4012	4637	1159	46	14.91	2312	34	1	1	625	1687
35	95	2375	4200	4825	1206	47	15.51	2450	35	25	5	25	1825
36	97	2425	4392	5017	1254	48	16.10	2592	36	1	1	625	1967
37	99	2475	4588	5213	1303	49	16.66	2738	37	1	1	625	2113
38	101	2525	4788	5413	1353	50	17.20	2888	38	1	1	625	2263
39	103	2575	4992	5617	1404	51	17.72	3042	39	1	1	625	2417
40	105	2625	5200	5825	1456	52	18.22	3200	40	25	5	25	2575

Table 25.2 Triples Sequences Step 1250: TS3; n(ν)=2ν+24; m=5·5

ν	n(ν)	x = 50n	y = n^2- 625	r = n^2+ 625	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	26	1300	51	1301	325		-42.75	1	1	1	1	1250	-1249
2	28	1400	159	1409	352	27	-38.52	9	3	1	1	1250	-1241
3	30	1500	275	1525	381	29	-34.61	25	5	25	5	50	-1225
4	32	1600	399	1649	412	31	-31.00	49	7	1	1	1250	-1201
5	34	1700	531	1781	445	33	-27.65	81	9	1	1	1250	-1169
6	36	1800	671	1921	480	35	-24.56	121	11	1	1	1250	-1129
7	38	1900	819	2069	517	37	-21.68	169	13	1	1	1250	-1081
8	40	2000	975	2225	556	39	-19.01	225	15	25	5	50	-1025
9	42	2100	1139	2389	597	41	-16.52	289	17	1	1	1250	-961
10	44	2200	1311	2561	640	43	-14.21	361	19	1	1	1250	-889
11	46	2300	1491	2741	685	45	-12.05	441	21	1	1	1250	-809
12	48	2400	1679	2929	732	47	-10.02	529	23	1	1	1250	-721
13	50	2500	1875	3125	781	49	-8.13	625	25	625	25	2	-625
14	52	2600	2079	3329	832	51	-6.35	729	27	1	1	1250	-521
15	54	2700	2291	3541	885	53	-4.68	841	29	1	1	1250	-409
16	56	2800	2511	3761	940	55	-3.11	961	31	1	1	1250	-289
17	58	2900	2739	3989	997	57	-1.63	1089	33	1	1	1250	-161
18	60	3000	2975	4225	1056	59	-0.24	1225	35	25	5	50	-25
19	62	**3100**	**3219**	**4469**	1117	61	**1.08**	1369	37	1	1	1250	119
20	64	3200	3471	4721	1180	63	2.33	1521	39	1	1	1250	271
21	66	3300	3731	4981	1245	65	3.51	1681	41	1	1	1250	431
22	68	3400	3999	5249	1312	67	4.63	1849	43	1	1	1250	599
23	70	3500	4275	5525	1381	69	5.69	2025	45	25	5	50	775
24	72	3600	4559	5809	1452	71	6.71	2209	47	1	1	1250	959
25	74	3700	4851	6101	1525	73	7.67	2401	49	1	1	1250	1151
26	76	3800	5151	6401	1600	75	8.58	2601	51	1	1	1250	1351
27	78	3900	5459	6709	1677	77	9.46	2809	53	1	1	1250	1559
28	80	4000	5775	7025	1756	79	10.29	3025	55	25	5	50	1775
29	82	4100	6099	7349	1837	81	11.09	3249	57	1	1	1250	1999
30	84	4200	6431	7681	1920	83	11.85	3481	59	1	1	1250	2231
31	86	4300	6771	8021	2005	85	12.58	3721	61	1	1	1250	2471
32	88	4400	7119	8369	2092	87	13.28	3969	63	1	1	1250	2719
33	90	4500	7475	8725	2181	89	13.95	4225	65	25	5	50	2975
34	92	4600	7839	9089	2272	91	14.60	4489	67	1	1	1250	3239
35	94	4700	8211	9461	2365	93	15.21	4761	69	1	1	1250	3511
36	96	4800	8591	9841	2460	95	15.81	5041	71	1	1	1250	3791
37	98	4900	8979	10229	2557	97	16.38	5329	73	1	1	1250	4079
38	100	5000	9375	10625	2656	99	16.93	5625	75	625	25	2	4375
39	102	5100	9779	11029	2757	101	17.46	5929	77	1	1	1250	4679
40	104	5200	10191	11441	2860	103	17.97	6241	79	1	1	1250	4991

Table 26 Triples Sequences Step 1352: TS2; $n(\nu)=2\nu+25$; $m=2\cdot 13$

ν	$n(\nu)$	$x = 52n$	$y = n^2 - 676$	$r = n^2 + 676$	$k = (r - 1)/4$	Δk	γ	$s_S = r - x$	m_S	α	β	$s_P = (r - y)/\alpha$	$y - x$
1	27	1404	53	1405	351		-42.84	1	1	1	1	1352	-1351
2	29	1508	165	1517	379	28	-38.76	9	3	1	1	1352	-1343
3	31	1612	285	1637	409	30	-34.97	25	5	1	1	1352	-1327
4	33	1716	413	1765	441	32	-31.47	49	7	1	1	1352	-1303
5	35	1820	549	1901	475	34	-28.21	81	9	1	1	1352	-1271
6	37	1924	693	2045	511	36	-25.19	121	11	1	1	1352	-1231
7	39	2028	845	2197	549	38	-22.38	169	13	169	13	8	-1183
8	41	2132	1005	2357	589	40	-19.76	225	15	1	1	1352	-1127
9	43	2236	1173	2525	631	42	-17.32	289	17	1	1	1352	-1063
10	45	2340	1349	2701	675	44	-15.04	361	19	1	1	1352	-991
11	47	2444	1533	2885	721	46	-12.90	441	21	1	1	1352	-911
12	49	2548	1725	3077	769	48	-10.90	529	23	1	1	1352	-823
13	51	2652	1925	3277	819	50	-9.02	625	25	1	1	1352	-727
14	53	2756	2133	3485	871	52	-7.26	729	27	1	1	1352	-623
15	55	2860	2349	3701	925	54	-5.60	841	29	1	1	1352	-511
16	57	2964	2573	3925	981	56	-4.04	961	31	1	1	1352	-391
17	59	3068	2805	4157	1039	58	-2.56	1089	33	1	1	1352	-263
18	61	*3172*	*3045*	*4397*	1099	60	-1.17	1225	35	1	1	1352	-127
19	63	**3276**	**3293**	**4645**	1161	62	**0.15**	1369	37	1	1	1352	17
20	65	3380	3549	4901	1225	64	1.40	1521	39	169	13	8	169
21	67	3484	3813	5165	1291	66	2.58	1681	41	1	1	1352	329
22	69	3588	4085	5437	1359	68	3.71	1849	43	1	1	1352	497
23	71	3692	4365	5717	1429	70	4.78	2025	45	1	1	1352	673
24	73	3796	4653	6005	1501	72	5.79	2209	47	1	1	1352	857
25	75	3900	4949	6301	1575	74	6.76	2401	49	1	1	1352	1049
26	77	4004	5253	6605	1651	76	7.69	2601	51	1	1	1352	1249
27	79	4108	5565	6917	1729	78	8.57	2809	53	1	1	1352	1457
28	81	4212	5885	7237	1809	80	9.41	3025	55	1	1	1352	1673
29	83	4316	6213	7565	1891	82	10.21	3249	57	1	1	1352	1897
30	85	4420	6549	7901	1975	84	10.99	3481	59	1	1	1352	2129
31	87	4524	6893	8245	2061	86	11.72	3721	61	1	1	1352	2369
32	89	4628	7245	8597	2149	88	12.43	3969	63	1	1	1352	2617
33	91	4732	7605	8957	2239	90	13.11	4225	65	169	13	8	2873
34	93	4836	7973	9325	2331	92	13.76	4489	67	1	1	1352	3137
35	95	4940	8349	9701	2425	94	14.39	4761	69	1	1	1352	3409
36	97	5044	8733	10085	2521	96	14.99	5041	71	1	1	1352	3689
37	99	5148	9125	10477	2619	98	15.57	5329	73	1	1	1352	3977
38	101	5252	9525	10877	2719	100	16.13	5625	75	1	1	1352	4273
39	103	5356	9933	11285	2821	102	16.67	5929	77	1	1	1352	4577
40	105	5460	10349	11701	2925	104	17.19	6241	79	1	1	1352	4889

Table 27.1 Triples Sequences Step 729: TS1; n(ν)=2ν+27; m=3·3·3

ν	n(ν)	x = 27n	y = $(n^2 - 729)/2$	r = $(n^2 + 729)/2$	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	29	783	56	785	196		-40.91	2	1	1	1	729	-727
2	31	837	116	845	211	15	-37.11	8	2	1	1	729	-721
3	33	891	180	909	227	16	-33.58	18	3	9	3	81	-711
4	35	945	248	977	244	17	-30.29	32	4	1	1	729	-697
5	37	999	320	1049	262	18	-27.24	50	5	1	1	729	-679
6	39	1053	396	1125	281	19	-24.39	72	6	9	3	81	-657
7	41	1107	476	1205	301	20	-21.73	98	7	1	1	729	-631
8	43	1161	560	1289	322	21	-19.25	128	8	1	1	729	-601
9	45	1215	648	1377	344	22	-16.93	162	9	81	9	9	-567
10	47	1269	740	1469	367	23	-14.75	200	10	1	1	729	-529
11	49	1323	836	1565	391	24	-12.71	242	11	1	1	729	-487
12	51	1377	936	1665	416	25	-10.79	288	12	9	3	81	-441
13	53	1431	1040	1769	442	26	-8.99	338	13	1	1	729	-391
14	55	1485	1148	1877	469	27	-7.29	392	14	1	1	729	-337
15	57	1539	1260	1989	497	28	-5.69	450	15	9	3	81	-279
16	59	1593	1376	2105	526	29	-4.18	512	16	1	1	729	-217
17	61	1647	1496	2225	556	30	-2.75	578	17	1	1	729	-151
18	63	1701	1620	2349	587	31	-1.40	648	18	81	9	9	-81
19	65	**1755**	**1748**	**2477**	619	32	**-0.11**	722	19	1	1	729	-7
20	67	1809	1880	2609	652	33	1.10	800	20	1	1	729	71
21	69	1863	2016	2745	686	34	2.26	882	21	9	3	81	153
22	71	1917	2156	2885	721	35	3.36	968	22	1	1	729	239
23	73	1971	2300	3029	757	36	4.41	1058	23	1	1	729	329
24	75	2025	2448	3177	794	37	5.40	1152	24	9	3	81	423
25	77	2079	2600	3329	832	38	6.36	1250	25	1	1	729	521
26	79	2133	2756	3485	871	39	7.26	1352	26	1	1	729	623
27	81	2187	2916	3645	911	40	8.13	1458	27	729	27	1	729
28	83	2241	3080	3809	952	41	8.96	1568	28	1	1	729	839
29	85	2295	3248	3977	994	42	9.76	1682	29	1	1	729	953
30	87	2349	3420	4149	1037	43	10.52	1800	30	9	3	81	1071
31	89	2403	3596	4325	1081	44	11.25	1922	31	1	1	729	1193
32	91	2457	3776	4505	1126	45	11.95	2048	32	1	1	729	1319
33	93	2511	3960	4689	1172	46	12.62	2178	33	9	3	81	1449
34	95	2565	4148	4877	1219	47	13.27	2312	34	1	1	729	1583
35	97	2619	4340	5069	1267	48	13.89	2450	35	1	1	729	1721
36	99	2673	4536	5265	1316	49	14.49	2592	36	81	9	9	1863
37	101	2727	4736	5465	1366	50	15.07	2738	37	1	1	729	2009
38	103	2781	4940	5669	1417	51	15.62	2888	38	1	1	729	2159
39	105	2835	5148	5877	1469	52	16.16	3042	39	9	3	81	2313
40	107	2889	5360	6089	1522	53	16.68	3200	40	1	1	729	2471

Table 27.2 Triples Sequences Step 1458: TS3; n(ν)=2ν+26; m=3·3·3

ν	n(ν)	x = 7n	y = n^2- 729	r = n^2+ 729	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	28	1512	55	1513	378		-42.92	1	1	1	1	1458	-1457
2	30	1620	171	1629	407	29	-38.97	9	3	9	3	162	-1449
3	32	1728	295	1753	438	31	-35.31	25	5	1	1	1458	-1433
4	34	1836	427	1885	471	33	-31.91	49	7	1	1	1458	-1409
5	36	1944	567	2025	506	35	-28.74	81	9	81	9	18	-1377
6	38	2052	715	2173	543	37	-25.79	121	11	1	1	1458	-1337
7	40	2160	871	2329	582	39	-23.04	169	13	1	1	1458	-1289
8	42	2268	1035	2493	623	41	-20.47	225	15	9	3	162	-1233
9	44	2376	1207	2665	666	43	-18.07	289	17	1	1	1458	-1169
10	46	2484	1387	2845	711	45	-15.82	361	19	1	1	1458	-1097
11	48	2592	1575	3033	758	47	-13.71	441	21	9	3	162	-1017
12	50	2700	1771	3229	807	49	-11.74	529	23	1	1	1458	-929
13	52	2808	1975	3433	858	51	-9.88	625	25	1	1	1458	-833
14	54	2916	2187	3645	911	53	-8.13	729	27	729	27	2	-729
15	56	3024	2407	3865	966	55	-6.48	841	29	1	1	1458	-617
16	58	3132	2635	4093	1023	57	-4.92	961	31	1	1	1458	-497
17	60	3240	2871	4329	1082	59	-3.45	1089	33	9	3	162	-369
18	62	3348	3115	4573	1143	61	-2.06	1225	35	1	1	1458	-233
19	64	**3456**	**3367**	**4825**	1206	63	**-0.75**	1369	37	1	1	1458	-89
20	66	3564	3627	5085	1271	65	0.50	1521	39	9	3	162	63
21	68	3672	3895	5353	1338	67	1.69	1681	41	1	1	1458	223
22	70	3780	4171	5629	1407	69	2.82	1849	43	1	1	1458	391
23	72	3888	4455	5913	1478	71	3.89	2025	45	81	9	18	567
24	74	3996	4747	6205	1551	73	4.91	2209	47	1	1	1458	751
25	76	4104	5047	6505	1626	75	5.88	2401	49	1	1	1458	943
26	78	4212	5355	6813	1703	77	6.81	2601	51	9	3	162	1143
27	80	4320	5671	7129	1782	79	7.70	2809	53	1	1	1458	1351
28	82	4428	5995	7453	1863	81	8.55	3025	55	1	1	1458	1567
29	84	4536	6327	7785	1946	83	9.36	3249	57	9	3	162	1791
30	86	4644	6667	8125	2031	85	10.14	3481	59	1	1	1458	2023
31	88	4752	7015	8473	2118	87	10.89	3721	61	1	1	1458	2263
32	90	4860	7371	8829	2207	89	11.60	3969	63	81	9	18	2511
33	92	4968	7735	9193	2298	91	12.29	4225	65	1	1	1458	2767
34	94	5076	8107	9565	2391	93	12.95	4489	67	1	1	1458	3031
35	96	5184	8487	9945	2486	95	13.58	4761	69	9	3	162	3303
36	98	5292	8875	10333	2583	97	14.19	5041	71	1	1	1458	3583
37	100	5400	9271	10729	2682	99	14.78	5329	73	1	1	1458	3871
38	102	5508	9675	11133	2783	101	15.35	5625	75	9	3	162	4167
39	104	5616	10087	11545	2886	103	15.89	5929	77	1	1	1458	4471
40	106	5724	10507	11965	2991	105	16.42	6241	79	1	1	1458	4783

Table 28 Triples Sequences Step 1568: TS2; n(ν)=2ν+27; m=2·2·7

ν	n(ν)	x = 56n	y = n^2- 784	r = n^2+ 784	k = (r - 1)/4	Δk	γ	s_S = r - x	m_S	α	β	s_P = (r - y)/α	y - x
1	29	1624	57	1625	406		-42.99	1	1	1	1	1568	-1567
2	31	1736	177	1745	436	30	-39.18	9	3	1	1	1568	-1559
3	33	1848	305	1873	468	32	-35.63	25	5	1	1	1568	-1543
4	35	1960	441	2009	502	34	-32.32	49	7	49	7	32	-1519
5	37	2072	585	2153	538	36	-29.23	81	9	1	1	1568	-1487
6	39	2184	737	2305	576	38	-26.35	121	11	1	1	1568	-1447
7	41	2296	897	2465	616	40	-23.66	169	13	1	1	1568	-1399
8	43	2408	1065	2633	658	42	-21.14	225	15	1	1	1568	-1343
9	45	2520	1241	2809	702	44	-18.78	289	17	1	1	1568	-1279
10	47	2632	1425	2993	748	46	-16.57	361	19	1	1	1568	-1207
11	49	2744	1617	3185	796	48	-14.49	441	21	49	7	32	-1127
12	51	2856	1817	3385	846	50	-12.53	529	23	1	1	1568	-1039
13	53	2968	2025	3593	898	52	-10.69	625	25	1	1	1568	-943
14	55	3080	2241	3809	952	54	-8.96	729	27	1	1	1568	-839
15	57	3192	2465	4033	1008	56	-7.32	841	29	1	1	1568	-727
16	59	3304	2697	4265	1066	58	-5.77	961	31	1	1	1568	-607
17	61	3416	2937	4505	1126	60	-4.31	1089	33	1	1	1568	-479
18	63	3528	3185	4753	1188	62	-2.92	1225	35	49	7	32	-343
19	65	3640	3441	5009	1252	64	-1.61	1369	37	1	1	1568	-199
20	67	**3752**	**3705**	**5273**	1318	66	**-0.36**	1521	39	1	1	1568	-47
21	69	3864	3977	5545	1386	68	0.83	1681	41	1	1	1568	113
22	71	3976	4257	5825	1456	70	1.96	1849	43	1	1	1568	281
23	73	4088	4545	6113	1528	72	3.03	2025	45	1	1	1568	457
24	75	4200	4841	6409	1602	74	4.06	2209	47	1	1	1568	641
25	77	4312	5145	6713	1678	76	5.04	2401	49	49	7	32	833
26	79	4424	5457	7025	1756	78	5.97	2601	51	1	1	1568	1033
27	81	4536	5777	7345	1836	80	6.86	2809	53	1	1	1568	1241
28	83	4648	6105	7673	1918	82	7.72	3025	55	1	1	1568	1457
29	85	4760	6441	8009	2002	84	8.54	3249	57	1	1	1568	1681
30	87	4872	6785	8353	2088	86	9.32	3481	59	1	1	1568	1913
31	89	4984	7137	8705	2176	88	10.07	3721	61	1	1	1568	2153
32	91	5096	7497	9065	2266	90	10.80	3969	63	49	7	32	2401
33	93	5208	7865	9433	2358	92	11.49	4225	65	1	1	1568	2657
34	95	5320	8241	9809	2452	94	12.16	4489	67	1	1	1568	2921
35	97	5432	8625	10193	2548	96	12.80	4761	69	1	1	1568	3193
36	99	5544	9017	10585	2646	98	13.42	5041	71	1	1	1568	3473
37	101	5656	9417	10985	2746	100	14.01	5329	73	1	1	1568	3761
38	103	5768	9825	11393	2848	102	14.59	5625	75	1	1	1568	4057
39	105	5880	10241	11809	2952	104	15.14	5929	77	49	7	32	4361
40	107	5992	10665	12233	3058	106	15.67	6241	79	1	1	1568	4673

Table 29.1 Triples Sequences Step 841: TS1; n(ν)=2ν+29; m=29

ν	n(ν)	x = 29n	y = (n^2- 841)/2	r = (n^2+ 841)/2	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	31	899	60	901	225		-41.18	2	1	1	1	841	-839
2	33	957	124	965	241	16	-37.62	8	2	1	1	841	-833
3	35	1015	192	1033	258	17	-34.29	18	3	1	1	841	-823
4	37	1073	264	1105	276	18	-31.18	32	4	1	1	841	-809
5	39	1131	340	1181	295	19	-28.27	50	5	1	1	841	-791
6	41	1189	420	1261	315	20	-25.54	72	6	1	1	841	-769
7	43	1247	504	1345	336	21	-22.99	98	7	1	1	841	-743
8	45	1305	592	1433	358	22	-20.60	128	8	1	1	841	-713
9	47	1363	684	1525	381	23	-18.35	162	9	1	1	841	-679
10	49	1421	780	1621	405	24	-16.24	200	10	1	1	841	-641
11	51	1479	880	1721	430	25	-14.25	242	11	1	1	841	-599
12	53	1537	984	1825	456	26	-12.37	288	12	1	1	841	-553
13	55	1595	1092	1933	483	27	-10.60	338	13	1	1	841	-503
14	57	1653	1204	2045	511	28	-8.93	392	14	1	1	841	-449
15	59	1711	1320	2161	540	29	-7.35	450	15	1	1	841	-391
16	61	1769	1440	2281	570	30	-5.85	512	16	1	1	841	-329
17	63	1827	1564	2405	601	31	-4.43	578	17	1	1	841	-263
18	65	1885	1692	2533	633	32	-3.09	648	18	1	1	841	-193
19	67	1943	1824	2665	666	33	-1.81	722	19	1	1	841	-119
20	69	2001	1960	2801	700	34	-0.59	800	20	1	1	841	-41
21	71	**2059**	**2100**	2941	735	35	**0.57**	882	21	1	1	841	41
22	73	2117	2244	3085	771	36	1.67	968	22	1	1	841	127
23	75	2175	2392	3233	808	37	2.72	1058	23	1	1	841	217
24	77	2233	2544	3385	846	38	3.73	1152	24	1	1	841	311
25	79	2291	2700	3541	885	39	4.69	1250	25	1	1	841	409
26	81	2349	2860	3701	925	40	5.60	1352	26	1	1	841	511
27	83	2407	3024	3865	966	41	6.48	1458	27	1	1	841	617
28	85	2465	3192	4033	1008	42	7.32	1568	28	1	1	841	727
29	87	2523	3364	4205	1051	43	8.13	1682	29	841	29	1	841
30	89	2581	3540	4381	1095	44	8.91	1800	30	1	1	841	959
31	91	2639	3720	4561	1140	45	9.65	1922	31	1	1	841	1081
32	93	2697	3904	4745	1186	46	10.36	2048	32	1	1	841	1207
33	95	2755	4092	4933	1233	47	11.05	2178	33	1	1	841	1337
34	97	2813	4284	5125	1281	48	11.71	2312	34	1	1	841	1471
35	99	2871	4480	5321	1330	49	12.35	2450	35	1	1	841	1609
36	101	2929	4680	5521	1380	50	12.96	2592	36	1	1	841	1751
37	103	2987	4884	5725	1431	51	13.55	2738	37	1	1	841	1897
38	105	3045	5092	5933	1483	52	14.12	2888	38	1	1	841	2047
39	107	3103	5304	6145	1536	53	14.67	3042	39	1	1	841	2201
40	109	3161	5520	6361	1590	54	15.20	3200	40	1	1	841	2359

Table 29.2 Triples Sequences Step 1682: TS3; n(ν)=2ν+28; m=29

ν	n(ν)	x = 58n	y = n^2- 841	r = n^2+ 841	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	30	1740	59	1741	435		-43.06	1	1	1	1	1682	-1681
2	32	1856	183	1865	466	31	-39.37	9	3	1	1	1682	-1673
3	34	1972	315	1997	499	33	-35.92	25	5	1	1	1682	-1657
4	36	2088	455	2137	534	35	-32.71	49	7	1	1	1682	-1633
5	38	2204	603	2285	571	37	-29.70	81	9	1	1	1682	-1601
6	40	2320	759	2441	610	39	-26.88	121	11	1	1	1682	-1561
7	42	2436	923	2605	651	41	-24.25	169	13	1	1	1682	-1513
8	44	2552	1095	2777	694	43	-21.78	225	15	1	1	1682	-1457
9	46	2668	1275	2957	739	45	-19.46	289	17	1	1	1682	-1393
10	48	2784	1463	3145	786	47	-17.28	361	19	1	1	1682	-1321
11	50	2900	1659	3341	835	49	-15.23	441	21	1	1	1682	-1241
12	52	3016	1863	3545	886	51	-13.30	529	23	1	1	1682	-1153
13	54	3132	2075	3757	939	53	-11.47	625	25	1	1	1682	-1057
14	56	3248	2295	3977	994	55	-9.75	729	27	1	1	1682	-953
15	58	3364	2523	4205	1051	57	-8.13	841	29	841	29	2	-841
16	60	3480	2759	4441	1110	59	-6.59	961	31	1	1	1682	-721
17	62	3596	3003	4685	1171	61	-5.13	1089	33	1	1	1682	-593
18	64	3712	3255	4937	1234	63	-3.75	1225	35	1	1	1682	-457
19	66	3828	3515	5197	1299	65	-2.44	1369	37	1	1	1682	-313
20	68	3944	3783	5465	1366	67	-1.19	1521	39	1	1	1682	-161
21	70	**4060**	**4059**	**5741**	1435	69	**-0.01**	1681	41	1	1	1682	-1
22	72	4176	4343	6025	1506	71	1.12	1849	43	1	1	1682	167
23	74	4292	4635	6317	1579	73	2.20	2025	45	1	1	1682	343
24	76	4408	4935	6617	1654	75	3.23	2209	47	1	1	1682	527
25	78	4524	5243	6925	1731	77	4.21	2401	49	1	1	1682	719
26	80	4640	5559	7241	1810	79	5.15	2601	51	1	1	1682	919
27	82	4756	5883	7565	1891	81	6.05	2809	53	1	1	1682	1127
28	84	4872	6215	7897	1974	83	6.91	3025	55	1	1	1682	1343
29	86	4988	6555	8237	2059	85	7.73	3249	57	1	1	1682	1567
30	88	5104	6903	8585	2146	87	8.52	3481	59	1	1	1682	1799
31	90	5220	7259	8941	2235	89	9.28	3721	61	1	1	1682	2039
32	92	5336	7623	9305	2326	91	10.01	3969	63	1	1	1682	2287
33	94	5452	7995	9677	2419	93	10.71	4225	65	1	1	1682	2543
34	96	5568	8375	10057	2514	95	11.38	4489	67	1	1	1682	2807
35	98	5684	8763	10445	2611	97	12.03	4761	69	1	1	1682	3079
36	100	5800	9159	10841	2710	99	12.66	5041	71	1	1	1682	3359
37	102	5916	9563	11245	2811	101	13.26	5329	73	1	1	1682	3647
38	104	6032	9975	11657	2914	103	13.84	5625	75	1	1	1682	3943
39	106	6148	10395	12077	3019	105	14.40	5929	77	1	1	1682	4247
40	108	6264	10823	12505	3126	107	14.94	6241	79	1	1	1682	4559

Table 30 Triples Sequences Step 1800: TS2; $n(\nu)=2\nu+29$; $m=2\cdot3\cdot5$

ν	$n(\nu)$	$x = 60n$	$y = n^2 - 900$	$r = n^2 + 900$	$k = (r - 1)/4$	Δk	γ	$s_s = r - x$	m_s	α	β	$s_p = (r - y)/\alpha$	y - x
1	31	1860	61	1861	465		-43.12	1	1	1	1	1800	-1799
2	33	1980	189	1989	497	32	-39.55	9	3	9	3	200	-1791
3	35	2100	325	2125	531	34	-36.20	25	5	25	5	72	-1775
4	37	2220	469	2269	567	36	-33.07	49	7	1	1	1800	-1751
5	39	2340	621	2421	605	38	-30.14	81	9	9	3	200	-1719
6	41	2460	781	2581	645	40	-27.39	121	11	1	1	1800	-1679
7	43	2580	949	2749	687	42	-24.80	169	13	1	1	1800	-1631
8	45	2700	1125	2925	731	44	-22.38	225	15	225	15	8	-1575
9	47	2820	1309	3109	777	46	-20.10	289	17	1	1	1800	-1511
10	49	2940	1501	3301	825	48	-17.95	361	19	1	1	1800	-1439
11	51	3060	1701	3501	875	50	-15.93	441	21	9	3	200	-1359
12	53	3180	1909	3709	927	52	-14.02	529	23	1	1	1800	-1271
13	55	3300	2125	3925	981	54	-12.22	625	25	25	5	72	-1175
14	57	3420	2349	4149	1037	56	-10.52	729	27	9	3	200	-1071
15	59	3540	2581	4381	1095	58	-8.90	841	29	1	1	1800	-959
16	61	3660	2821	4621	1155	60	-7.38	961	31	1	1	1800	-839
17	63	3780	3069	4869	1217	62	-5.93	1089	33	9	3	200	-711
18	65	3900	3325	5125	1281	64	-4.55	1225	35	25	5	72	-575
19	67	4020	3589	5389	1347	66	-3.24	1369	37	1	1	1800	-431
20	69	4140	3861	5661	1415	68	-2.00	1521	39	9	3	200	-279
21	71	4260	4141	5941	1485	70	-0.81	1681	41	1	1	1800	-119
22	73	**4380**	**4429**	**6229**	1557	72	**0.32**	1849	43	1	1	1800	49
23	75	4500	4725	6525	1631	74	1.40	2025	45	225	15	8	225
24	77	4620	5029	6829	1707	76	2.43	2209	47	1	1	1800	409
25	79	4740	5341	7141	1785	78	3.41	2401	49	1	1	1800	601
26	81	4860	5661	7461	1865	80	4.36	2601	51	9	3	200	801
27	83	4980	5989	7789	1947	82	5.26	2809	53	1	1	1800	1009
28	85	5100	6325	8125	2031	84	6.12	3025	55	25	5	72	1225
29	87	5220	6669	8469	2117	86	6.95	3249	57	9	3	200	1449
30	89	5340	7021	8821	2205	88	7.75	3481	59	1	1	1800	1681
31	91	5460	7381	9181	2295	90	8.51	3721	61	1	1	1800	1921
32	93	5580	7749	9549	2387	92	9.24	3969	63	9	3	200	2169
33	95	5700	8125	9925	2481	94	9.95	4225	65	25	5	72	2425
34	97	5820	8509	10309	2577	96	10.63	4489	67	1	1	1800	2689
35	99	5940	8901	10701	2675	98	11.28	4761	69	9	3	200	2961
36	101	6060	9301	11101	2775	100	11.92	5041	71	1	1	1800	3241
37	103	6180	9709	11509	2877	102	12.52	5329	73	1	1	1800	3529
38	105	6300	10125	11925	2981	104	13.11	5625	75	225	15	8	3825
39	107	6420	10549	12349	3087	106	13.68	5929	77	1	1	1800	4129
40	109	6540	10981	12781	3195	108	14.22	6241	79	1	1	1800	4441

Table 31.1 Triples Sequences Step 961: TS1; $n(\nu)=2\nu+31$; m=31

ν	$n(\nu)$	$x = 31n$	$y = n^2 - 961$	$r = n^2 + 961$	$k = (r - 1)/4$	Δk	γ	$s_S = r - x$	m_S	α	β	$s_P = (r - y)/\alpha$	y - x
1	33	1023	64	1025	256		-41.42	2	1	1	1	961	-959
2	35	1085	132	1093	273	17	-38.06	8	2	1	1	961	-953
3	37	1147	204	1165	291	18	-34.91	18	3	1	1	961	-943
4	39	1209	280	1241	310	19	-31.96	32	4	1	1	961	-929
5	41	1271	360	1321	330	20	-29.19	50	5	1	1	961	-911
6	43	1333	444	1405	351	21	-26.58	72	6	1	1	961	-889
7	45	1395	532	1493	373	22	-24.12	98	7	1	1	961	-863
8	47	1457	624	1585	396	23	-21.81	128	8	1	1	961	-833
9	49	1519	720	1681	420	24	-19.64	162	9	1	1	961	-799
10	51	1581	820	1781	445	25	-17.59	200	10	1	1	961	-761
11	53	1643	924	1885	471	26	-15.65	242	11	1	1	961	-719
12	55	1705	1032	1993	498	27	-13.81	288	12	1	1	961	-673
13	57	1767	1144	2105	526	28	-12.08	338	13	1	1	961	-623
14	59	1829	1260	2221	555	29	-10.44	392	14	1	1	961	-569
15	61	1891	1380	2341	585	30	-8.88	450	15	1	1	961	-511
16	63	1953	1504	2465	616	31	-7.40	512	16	1	1	961	-449
17	65	2015	1632	2593	648	32	-5.99	578	17	1	1	961	-383
18	67	2077	1764	2725	681	33	-4.66	648	18	1	1	961	-313
19	69	2139	1900	2861	715	34	-3.39	722	19	1	1	961	-239
20	71	2201	2040	3001	750	35	-2.17	800	20	1	1	961	-161
21	73	2263	2184	3145	786	36	-1.02	882	21	1	1	961	-79
22	75	**2325**	**2332**	**3293**	823	37	**0.09**	968	22	1	1	961	7
23	77	2387	2484	3445	861	38	1.14	1058	23	1	1	961	97
24	79	2449	2640	3601	900	39	2.15	1152	24	1	1	961	191
25	81	2511	2800	3761	940	40	3.12	1250	25	1	1	961	289
26	83	2573	2964	3925	981	41	4.04	1352	26	1	1	961	391
27	85	2635	3132	4093	1023	42	4.93	1458	27	1	1	961	497
28	87	2697	3304	4265	1066	43	5.78	1568	28	1	1	961	607
29	89	2759	3480	4441	1110	44	6.59	1682	29	1	1	961	721
30	91	2821	3660	4621	1155	45	7.38	1800	30	1	1	961	839
31	93	2883	3844	4805	1201	46	8.13	1922	31	961	31	1	961
32	95	2945	4032	4993	1248	47	8.86	2048	32	1	1	961	1087
33	97	3007	4224	5185	1296	48	9.56	2178	33	1	1	961	1217
34	99	3069	4420	5381	1345	49	10.23	2312	34	1	1	961	1351
35	101	3131	4620	5581	1395	50	10.88	2450	35	1	1	961	1489
36	103	3193	4824	5785	1446	51	11.50	2592	36	1	1	961	1631
37	105	3255	5032	5993	1498	52	12.10	2738	37	1	1	961	1777
38	107	3317	5244	6205	1551	53	12.69	2888	38	1	1	961	1927
39	109	3379	5460	6421	1605	54	13.25	3042	39	1	1	961	2081
40	111	3441	5680	6641	1660	55	13.79	3200	40	1	1	961	2239

Table 31.2 Triples Sequences Step 1922: TS3; $n(\nu)=2\nu+30$; m=31

ν	$n(\nu)$	x = 961n	y = n^2- 961	r = n^2+ 962	k = (r - 1)/4	Δk	γ	s_s = r - x	m_s	α	β	s_p = (r - y)/α	y - x
1	32	1984	63	1985	496		-43.18	1	1	1	1	1922	-1921
2	34	2108	195	2117	529	33	-39.71	9	3	1	1	1922	-1913
3	36	2232	335	2257	564	35	-36.46	25	5	1	1	1922	-1897
4	38	2356	483	2405	601	37	-33.41	49	7	1	1	1922	-1873
5	40	2480	639	2561	640	39	-30.55	81	9	1	1	1922	-1841
6	42	2604	803	2725	681	41	-27.86	121	11	1	1	1922	-1801
7	44	2728	975	2897	724	43	-25.33	169	13	1	1	1922	-1753
8	46	2852	1155	3077	769	45	-22.95	225	15	1	1	1922	-1697
9	48	2976	1343	3265	816	47	-20.71	289	17	1	1	1922	-1633
10	50	3100	1539	3461	865	49	-18.60	361	19	1	1	1922	-1561
11	52	3224	1743	3665	916	51	-16.60	441	21	1	1	1922	-1481
12	54	3348	1955	3877	969	53	-14.72	529	23	1	1	1922	-1393
13	56	3472	2175	4097	1024	55	-12.93	625	25	1	1	1922	-1297
14	58	3596	2403	4325	1081	57	-11.25	729	27	1	1	1922	-1193
15	60	3720	2639	4561	1140	59	-9.65	841	29	1	1	1922	-1081
16	62	3844	2883	4805	1201	61	-8.13	961	31	961	31	2	-961
17	64	3968	3135	5057	1264	63	-6.69	1089	33	1	1	1922	-833
18	66	4092	3395	5317	1329	65	-5.32	1225	35	1	1	1922	-697
19	68	4216	3663	5585	1396	67	-4.01	1369	37	1	1	1922	-553
20	70	4340	3939	5861	1465	69	-2.77	1521	39	1	1	1922	-401
21	72	4464	4223	6145	1536	71	-1.59	1681	41	1	1	1922	-241
22	74	**4588**	**4515**	**6437**	1609	73	**-0.46**	1849	43	1	1	1922	-73
23	76	4712	4815	6737	1684	75	0.62	2025	45	1	1	1922	103
24	78	4836	5123	7045	1761	77	1.65	2209	47	1	1	1922	287
25	80	4960	5439	7361	1840	79	2.64	2401	49	1	1	1922	479
26	82	5084	5763	7685	1921	81	3.58	2601	51	1	1	1922	679
27	84	5208	6095	8017	2004	83	4.49	2809	53	1	1	1922	887
28	86	5332	6435	8357	2089	85	5.36	3025	55	1	1	1922	1103
29	88	5456	6783	8705	2176	87	6.19	3249	57	1	1	1922	1327
30	90	5580	7139	9061	2265	89	6.99	3481	59	1	1	1922	1559
31	92	5704	7503	9425	2356	91	7.76	3721	61	1	1	1922	1799
32	94	5828	7875	9797	2449	93	8.50	3969	63	1	1	1922	2047
33	96	5952	8255	10177	2544	95	9.21	4225	65	1	1	1922	2303
34	98	6076	8643	10565	2641	97	9.89	4489	67	1	1	1922	2567
35	100	6200	9039	10961	2740	99	10.55	4761	69	1	1	1922	2839
36	102	6324	9443	11365	2841	101	11.19	5041	71	1	1	1922	3119
37	104	6448	9855	11777	2944	103	11.81	5329	73	1	1	1922	3407
38	106	6572	10275	12197	3049	105	12.40	5625	75	1	1	1922	3703
39	108	6696	10703	12625	3156	107	12.97	5929	77	1	1	1922	4007
40	110	6820	11139	13061	3265	109	13.52	6241	79	1	1	1922	4319

Table 32 Triples Sequences Step 2048: TS2; $n(\nu)=2\nu+31$; $m=2\cdot2\cdot2\cdot2\cdot2$

ν	$n(\nu)$	$x = 64n$	$y = n^2 - 1024$	$r = n^2 + 1024$	$k = (r - 1)/4$	Δk	γ	$s_S = r - x$	m_S	α	β	$s_P = (r - y)/\alpha$	$y - x$
1	33	2112	65	2113	528		-43.24	1	1	1	1	2048	-2047
2	35	2240	201	2249	562	34	-39.87	9	3	1	1	2048	-2039
3	37	2368	345	2393	598	36	-36.71	25	5	1	1	2048	-2023
4	39	2496	497	2545	636	38	-33.74	49	7	1	1	2048	-1999
5	41	2624	657	2705	676	40	-30.94	81	9	1	1	2048	-1967
6	43	2752	825	2873	718	42	-28.31	121	11	1	1	2048	-1927
7	45	2880	1001	3049	762	44	-25.83	169	13	1	1	2048	-1879
8	47	3008	1185	3233	808	46	-23.50	225	15	1	1	2048	-1823
9	49	3136	1377	3425	856	48	-21.29	289	17	1	1	2048	-1759
10	51	3264	1577	3625	906	50	-19.21	361	19	1	1	2048	-1687
11	53	3392	1785	3833	958	52	-17.24	441	21	1	1	2048	-1607
12	55	3520	2001	4049	1012	54	-15.38	529	23	1	1	2048	-1519
13	57	3648	2225	4273	1068	56	-13.62	625	25	1	1	2048	-1423
14	59	3776	2457	4505	1126	58	-11.95	729	27	1	1	2048	-1319
15	61	3904	2697	4745	1186	60	-10.36	841	29	1	1	2048	-1207
16	63	4032	2945	4993	1248	62	-8.85	961	31	1	1	2048	-1087
17	65	4160	3201	5249	1312	64	-7.42	1089	33	1	1	2048	-959
18	67	4288	3465	5513	1378	66	-6.06	1225	35	1	1	2048	-823
19	69	4416	3737	5785	1446	68	-4.76	1369	37	1	1	2048	-679
20	71	4544	4017	6065	1516	70	-3.52	1521	39	1	1	2048	-527
21	73	4672	4305	6353	1588	72	-2.34	1681	41	1	1	2048	-367
22	75	4800	4601	6649	1662	74	-1.21	1849	43	1	1	2048	-199
23	77	**4928**	**4905**	**6953**	1738	76	**-0.13**	2025	45	1	1	2048	-23
24	79	5056	5217	7265	1816	78	0.90	2209	47	1	1	2048	161
25	81	5184	5537	7585	1896	80	1.89	2401	49	1	1	2048	353
26	83	5312	5865	7913	1978	82	2.83	2601	51	1	1	2048	553
27	85	5440	6201	8249	2062	84	3.74	2809	53	1	1	2048	761
28	87	5568	6545	8593	2148	86	4.61	3025	55	1	1	2048	977
29	89	5696	6897	8945	2236	88	5.45	3249	57	1	1	2048	1201
30	91	5824	7257	9305	2326	90	6.25	3481	59	1	1	2048	1433
31	93	5952	7625	9673	2418	92	7.03	3721	61	1	1	2048	1673
32	95	6080	8001	10049	2512	94	7.77	3969	63	1	1	2048	1921
33	97	6208	8385	10433	2608	96	8.49	4225	65	1	1	2048	2177
34	99	6336	8777	10825	2706	98	9.18	4489	67	1	1	2048	2441
35	101	6464	9177	11225	2806	100	9.84	4761	69	1	1	2048	2713
36	103	6592	9585	11633	2908	102	10.48	5041	71	1	1	2048	2993
37	105	6720	10001	12049	3012	104	11.10	5329	73	1	1	2048	3281
38	107	6848	10425	12473	3118	106	11.70	5625	75	1	1	2048	3577
39	109	6976	10857	12905	3226	108	12.28	5929	77	1	1	2048	3881
40	111	7104	11297	13345	3336	110	12.84	6241	79	1	1	2048	4193

Table RPC1.1 Fermat Primes and Fermat Composites From r(m, n) In Tables 1-32

νr	νp	νc	r(νr)	Upper Table	c(νc)	νr	νp	νc	r(νr)	Upper Table	c(νc)
1	1		5	1		41		12	305	4, 9	5 · 61
2	2		13	1		42	30		313	1	
3	3		17	1		43	31		317	3	
4		1	25	1	5 • 5	44		13	325	1, 6	5 · 5 · 13
5	4		29	2		45	32		337	7	
6	5		37	1		46	33		349	5	
7	6		41	1		47	34		353	9	
8	7		53	2		48		14	365	1, 2	5 · 73
9	8		61	1		49	35		373	7	
10		2	65	1, 3	5 • 13	50		15	377	4, 5	13 • 29
11	9		73	3		51	36		389	7	
12		3	85	1, 2	5 · 17	52	37		397	6	
13	10		89	3		53	38		401	1	
14	11		97	5		54	39		409	3	
15	12		101	1		55	40		421	1	
16	13		109	3		56		16	425	3, 11	5 • 5 • 17
17	14		113	1		57	41		433	5	
18		4	125	2	5 • 5 • 5	58		17	445	2, 7	5 • 89
19	15		137	4		59	42		449	7	
20		5	145	1, 1	5 · 29	60	43		457	4	
21	16		149	3		61	44		461	9	
22	17		157	5		62		18	481	1, 11	13 • 37
23		6	169	7	13 · 13	63		19	485	1, 3	5 • 97
24	18		173	2		64		20	493	3, 5	17 • 29
25	19		181	1		65		21	505	7, 8	5 • 101
26		7	185	3, 4	5 • 37	66	45		509	5	
27	20		193	5		67	46		521	9	
28	21		197	1		68		22	533	2, 7	13 • 41
29		8	205	3, 7	5 • 41	69	47		541	11	
30		9	221	1, 5	13 • 17	70		23	545	1, 4	5 • 109
31	22		229	2		71	48		557	5	
32	23		233	5		72		24	565	6, 9	5 • 113
33	24		241	4		73	49		569	7	
34	25		257	1		74	50		577	1	
35		10	265	1, 3	5 • 53	75	51		593	8	
36	26		269	3		76	52		601	5	
37	27		277	5		77	53		613	1	
38	28		281	5		78	54		617	3	
39		11	289	7	17 • 17	79		25	625	7	5 • 5 • 5 • 5
40	29		293	2		80		26	629	2, 13	17 • 37

Table RPC1.2 Fermat Primes and Fermat Composites From r(m, n) In Tables 1-32

νr	νp	νc	r(νr)	Upper Table	c(νc)	νr	νp	νc	r(νr)	Upper Table	c(νc)
81	55		641	4		121		42	985	11, 12	5 • 197
82	56		653	9		122	80		997	6	
83	57		661	6		123	81		1009	12	
84	58		673	11		124	82		1013	1	
85	59		677	1		125	83		1021	11	
86		27	685	1, 3	5 • 137	126		43	1025	1, 8	5 • 5 • 41
87		28	689	3, 8	13 · 53	127	84		1033	3	
88		29	697	5, 13	17 · 41	128		44	1037	7, 15	17 · 61
89	60		701	5		129	85		1049	5	
90	61		709	7		130	86		1061	10	
91		30	725	7, 9	5 · 5 · 29	131	87		1069	17	
92	62		733	2		132		45	1073	7, 11	29 • 37
93		31	745	4, 11	5 · 149	133	88		1093	2	
94	63		757	9		134	89		1097	13	
95	64		761	1		135		46	1105	1, 4, 9, 12	5 • 13 • 17
96	65		769	13		136	90		1109	3	
97	66		773	5		137	91		1117	5	
98		32	785	1, 7	5 · 157	138	92		1129	7	
99		33	793	3, 8	13 · 61	139		47	1145	9, 11	5 · 229
100	67		797	15		140	93		1153	8	
101	68		809	5		141		48	1157	1, 17	13 • 89
102	69		821	11		142		49	1165	3, 11	5 • 233
103	70		829	10		143	94		1181	5	
104		34	841	1	29 · 29	144		50	1189	10, 13	29 · 41
105		35	845	2, 3	5 · 13 · 13	145	95		1193	13	
106	71		853	5		146	96		1201	1	
107	72		857	4		147		51	1205	3, 7	5 • 241
108		36	865	7, 9	5 · 173	148	97		1213	5	
109	73		877	6		149	98		1217	15	
110	74		881	9		150	99		1229	2	
111		37	901	1, 11	17 · 53	151	100		1237	9	
112		38	905	8, 11	5 · 181	152		52	1241	4, 9	17 • 73
113		39	925	1, 13	5 · 5 · 37	153	101		1249	17	
114	75		929	3		154		53	1261	6, 11	13 • 97
115	76		937	5		155	102		1277	11	
116	77		941	10		156		54	1285	13, 19	5 • 257
117		40	949	7, 7	13 · 73	157	103		1289	8	
118	78		953	15		158	104		1297	1	
119		41	965	2, 9	5 · 193	159	105		1301	1	
120	79		977	4		160		55	1313	5, 15	13 • 101

Table RPC1.3 Fermat Primes and Fermat Composites From r(m, n) In Tables 1-32

ν_r	ν_p	ν_c	$r(\nu_r)$	Upper Table	$c(\nu_c)$	ν_r	ν_p	ν_c	$r(\nu_r)$	Upper Table	$c(\nu_c)$
161	106		1321	5		201	127		1693	19	
162		56	1325	7, 13	5 · 5 · 53	202	128		1697	4	
163		57	1345	7, 17	5 · 269	203	129		1709	13	
164	107		1361	12		204		75	1717	6, 14	17 · 101
165		58	1369	12	37 · 37	205	130		1721	11	
166	108		1373	2		206	131		1733	21	
167	109		1381	19		207	132		1741	1	
168		59	1385	4, 13	5 · 277	208		76	1745	3, 8	5 · 349
169		60	1405	1, 6	5 · 281	209	133		1753	5	
170	110		1409	3		210		77	1765	1, 7	5 · 353
171		61	1417	5, 11	13 · 109	211		78	1769	13, 17	29 · 61
172	111		1429	7		212	134		1777	16	
173	112		1433	8		213		79	1781	9, 10	13 · 137
174		62	1445	1, 9	5 · 17 · 17	214	135		1789	5	
175	113		1453	3		215	136		1801	11	
176		63	1465	11, 13	5 · 293	216		80	1825	12, 13	5 · 5 · 73
177		64	1469	5, 10	13 · 113	217		81	1853	2, 15	17 · 109
178	114		1481	19		218	137		1861	1	
179	115		1489	13		219		82	1865	3, 4	5 · 373
180	116		1493	7		220	138		1873	5	
181		65	1513	1, 12	17 · 89	221	139		1877	14	
182		66	1517	3, 15	37 · 41	222		83	1885	6, 7, 11, 17	5 · 13 · 29
183		67	1525	2, 9	5 · 5 · 61	223	140		1889	23	
184		68	1537	4, 7	29 · 53	224	141		1901	9	
185	117		1549	17		225	142		1913	8	
186	118		1553	9		226		84	1921	11, 19	17 · 113
187		69	1565	11, 14	5 · 313	227	143		1933	13	
188		70	1585	8, 19	5 · 317	228		85	1937	1, 16	13 · 149
189	119		1597	13		229		86	1945	3, 13	5 · 389
190	120		1601	1		230	144		1949	10	
191	121		1609	3		231		87	1961	5, 21	37 · 53
192	122		1613	13		232	145		1973	15	
193	123		1621	10		233		88	1985	1, 7	5 · 397
194		71	1625	1, 21	5 · 5 · 5 · 13	234	146		1993	12	
195	124		1637	5		235	147		1997	5	
196		72	1649	7, 7	17 · 97	236		89	2005	17,23	5 · 401
197	125		1657	17		237	148		2017	9	
198	126		1669	15		238	149		2029	2	
199		73	1681	9	41 · 41	239		90	2041	4, 19	13 · 157
200		74	1685	2, 11	5 · 337	240		91	2045	11, 12	5 · 409

Table RPC1.4 Fermat Primes and Fermat Composites From r(m, n) In Tables 1-32

νr	νp	νc	r(νr)	Upper Table	c(νc)	νr	νp	νc	r(νr)	Upper Table	c(νc)
241	150		2053	17		281		106	2405	2, 7, 14, 17	5 · 13 · 37
242	151		2069	13		282	176		2417	4	
243	152		2081	21		283		107	2425	11, 19	5 · 5 · 97
244	153		2089	8		284	177		2437	6	
245		92	2105	13, 16	5 · 421	285	178		2441	11	
246	154		2113	1		286		108	2465	8, 13, 16, 21	5 · 17 · 29
247		93	2117	1, 3	29 · 73	287	179		2473	13	
248		94	2125	3, 25	5 · 5 · 5 · 17	288	180		2477	19	
249	155		2129	17		289		109	2501	1, 10	41 · 61
250	156		2137	7		290		110	2509	3, 23	13 · 193
251	157		2141	5		291	181		2521	1	
252	158		2153	9		292		111	2525	3, 17	5 · 5 · 101
253	159		2161	15		293		112	2533	5, 18	17 · 149
254		95	2165	7, 19	5 · 433	294		113	2545	7, 12	5 · 509
255		96	2173	11, 25	41 · 53	295	182		2549	7	
256		97	2197	9	13 · 13 · 13	296	183		2557	25	
257	160		2213	2		297		114	2561	9, 19	13 · 197
258	161		2221	14		298		115	2581	9, 11	29 · 89
259		98	2225	4, 17	5 · 5 · 89	299	184		2593	17	
260	162		2237	11		300		116	2605	2, 13	5 · 521
261		99	2245	1, 6	5 · 449	301	185		2609	27	
262		100	2249	3, 23	13 · 173	302	186		2617	4	
263		101	2257	5, 17	37 · 61	303	187		2621	11	
264	163		2269	7		304	188		2633	15	
265	164		2273	8		305	189		2657	16	
266	165		2281	1		306		117	2665	1, 8, 17, 19	5 · 13 · 41
267		102	2285	9, 13	5 · 457	307		118	2669	3, 13	17 · 157
268	166		2293	19		308	190		2677	5	
269	167		2297	25		309	191		2689	7	
270		103	2305	1, 11	5 · 461	310	192		2693	25	
271	168		2309	10		311		119	2701	10, 19	37 · 73
272		104	2329	5, 13	17 · 137	312		120	2705	1, 9	5 · 541
273	169		2333	21		313	193		2713	3	
274	170		2341	15		314		121	2725	11, 18	5 · 5 · 109
275		105	2353	7, 12	13 · 181	315	194		2729	5	
276	171		2357	15		316	195		2741	21	
277	172		2377	23		317	196		2749	13	
278	173		2381	1		318	197		2753	7	
279	174		2389	17		319	198		2777	15	
280	175		2393	5		320		122	2785	9, 23	5 · 557

Table RPC1.5 Fermat Primes and Fermat Composites From r(m, n) In Tables 1-32

ν_r	ν_p	ν_c	$r(\nu_r)$	Upper Table	$c(\nu_c)$	ν_r	ν_p	ν_c	$r(\nu_r)$	Upper Table	$c(\nu_c)$
321	199		2789	17		333		128	2885	21, 27	5 · 577
322	200		2797	14		334	206		2897	13	
323	201		2801	20		335	207		2909	10	
324		123	2809	17	53 · 53	336	208		2917	1	
325		124	2813	1, 2	29 · 97	337		129	2929	15, 23	29 · 101
326		125	2825	4, 11	5 · 5 · 113	338		130	2941	5, 29	17 · 173
327	202		2833	25		339	209		2953	12	
328	203		2837	7		340	210		2957	17	
329		126	2845	6, 19	5 · 569	341		131	2965	1, 7	5 · 593
330	204		2857	16		342	211		2969	3	
331	205		2861	19		343		132	2977	5, 25	13 · 229
332		127	2873	8, 11	13 · 13 · 17	344		133	2993	17, 19	41 · 73

Table RPC2 Unevaluated r(m, n) of U/L Triples From Table RPC1

row	νr	νp	r	5	Table	Triples	U/L	c(νc)	m	n	β	r(m, n)	TSj
1	1	1		5	1.1	{3, 4, 5}	U		1	3	1	(3^2 + 1)/2	TS1
2	1	1		5	1.2	{4, 3, 5)	L		1	2	1	2^2 + 1	TS3
3	2	2		13	1.1	{5, 12, 13}	U		1	5	1	(5^2 + 1)/2	TS1
4	2	2		13	2	{12, 5, 13}	L		2	3	1	3^2 + 2^2	TS2
5	3	3		17	1.2	{8, 15, 17	U		1	4	1	4^2 + 1	TS3
6	3	3		17	3.1	{15, 8, 17}	L		3	5	1	(5^2 + 3^2)/2	TS1
7	4		1	25	1.1	{7, 24, 25}	U	5·5	1	7	1	(7^2 + 1)/2	TS1
8	4		1	25	3.2	{24, 7, 25}	L	5·5	3	4	1	4^2 + 3^2	TS3
9	5	4		29	2	{20, 21, 29}	U		2	5	1	5^2 + 2^2	TS2
10	5	4		29	3.1	{21, 20, 29}	L		3	7	1	(7^2 + 3^2)/2	TS1
11	6	5		37	1.2	{12, 35, 37}	U		1	6	1	6^2 + 1	TS3
12	6	5		37	5.1	{35, 12, 37}	L		5	7	1	(7^2 + 5^2)/2	TS1
13	7	6		41	1.1	{9, 40, 41}	U		1	9	1	(9^2 + 1)/2	TS1
14	7	6		41	4	{40, 9, 41}	L		4	5	1	5^2 + 4^2	TS2
15	8	7		53	2	{28, 45, 53}	U		2	7	1	7^2 + 2^2	TS2
16	8	7		53	5.1	{45, 28, 53}	L		5	9	1	(9^2 + 5^2)/2	TS1
17	9	8		61	1.1	{11, 60, 61}	U		1	11	1	(11^2 + 1)/2	TS1
18	9	8		61	5.2	{60, 11, 61}	L		5	6	1	6^2 + 5^2	TS3
19	10		2	65	1.2	{16, 63, 65}	U	5·13	1	8	1	8^2 + 1	TS3
20	10		2	65	7.1	{63, 16, 65}	L	5·13	7	9	1	(9^2 + 7^2)/2	TS1
21	10		2	65	3.1	{33, 56, 65}	U	5·13	3	11	1	(11^2 + 3^2)/2	TS1
22	10		2	65	4	{56, 33, 65}	L	5·13	4	7	1	7^2 + 4^2	TS2
23	11	9		73	3.2	{48, 55, 73}	U		3	8	1	8^2 + 3^2	TS3
24	11	9		73	5.1	{55, 48, 73}	L		5	11	1	(11^2 + 5^2)/2	TS1
25	12		3	85	1.1	{13, 84, 85}	U	5·17	1	13	1	(13^2 + 1)/2	TS1
26	12		3	85	6	{84, 13, 85}	L	5·17	6	7	1	7^2 + 6^2	TS2
27	12		3	85	2	{36, 77, 85}	U	5·17	2	9	1	9^ 2+ 2^2	TS2
28	12		3	85	7.1	{77, 36, 85}	L	5·17	7	11	1	(11^2 + 7^2)/2	TS1
29	13	10		89	3.1	{39, 80, 89}	U		3	13	1	(13^2 + 3^2)/2	TS1
30	13	10		89	5.2	{80, 39, 89}	L		5	8	1	8^2 + 5^2	TS3
31	14	11		97	5.1	{65, 72, 97}	U		5	13	1	(13^2 + 5^2)/2	TS1
32	14	11		97	4	{72, 65, 97}	L		4	9	1	9^2 + 4^2	TS2
33	15	12		101	1.2	{20, 99, 101}	U		1	10	1	10^2 + 1	TS3
34	15	12		101	9.1	{99, 20, 101}	L		9	11	1	(11^2 + 9^2)/2	TS1
35	16	13		109	3.2	(60, 91, 109}	U		3	10	1	10^2 + 3^2	TS3
36	16	13		109	7.1	{91, 60, 109}	L		7	13	1	(13^2 + 7^2)/2	TS1
37	17	14		113	1.1	{15, 112, 113}	U		1	15	1	(15^2 + 1)/2	TS1
38	17	14		113	7.2	{112, 15, 113}	L		7	8	1	8^2 + 7^2	TS3
39	18		4	125	2	{44, 117, 125}	U	5·5·5	2	11	1	11^2 + 2^2	TS2
40	18		4	125	9.1	{117, 44, 125}	L	5·5·5	9	13	1	(13^2 + 9^2)/2	TS1

Table RPC3 Primitive Triple Factors From Unevaluated r(m, n)

row	ν_r	r	Table	Triple	U/L	α	Composite	m	n	r(m, n) –> α · (PT Factor)	r_{PT}
1	1	45	3.1	{27, 36, 45}	U	9	3 · 3 · 5	3	9	$(9^2 + 3^2)/2$ –> $9(3^2 + 1)/2$	5
2	1	45	3.2	{36, 27, 45}	L	9	3 · 3 · 5	3	6	$6^2 + 3^2$ –> $9(2^2 +1)$	5
3	2	117	3.1	(45, 108, 117}	U	9	3 · 3 · 13	3	15	$(15^2 + 3^2)/2$ –> $9(5^2 + 1)/2$	13
4	2	117	6	{108, 45, 117}	L	9	3 · 3 · 13	6	9	$9^2 + 6^2$ –> $9(3^2 + 2^2)$	13
5	3	125	5.1	{75, 100, 125}	U	25	5 · 5 · 5	5	15	$(15^2 + 5^2)/2$ –> $25(3^2 + 1)/2$	5
6	3	125	5.2	{100, 75, 125}	L	25	5 · 5 · 5	5	10	$10^2 + 5$ –> $25(2^2 + 1)$	5
7	4	153	3.2	{72, 135, 153}	U	9	3 · 3 · 17	3	12	$12^2 + 3^2$ –> $9(4^2 + 1)$	17
8	4	153	9.1	{135, 72, 153}	L	9	3 · 3 · 17	9	15	$(15^2 + 9^2)/2$ –> $9(5^2 + 3^2)/2$	17
9	5	225	3.1	{63, 216, 225}	U	9	3 · 3 · 25	3	21	$(21^2 + 3^2)/2$ –> $9(7^2 + 1)/2$	25
10	5	225	9.2	{216, 63, 225}	L	9	3 · 3 · 25	9	12	$12^2 + 9^2$ –> $9(4^2 + 3^2)$	25
11	6	245	7.1	{147, 196, 245}	U	49	7 · 7 · 5	7	21	$(21^2 + 7^2)/2$ –> $49(3^2 + 1)/2$	5
12	6	245	7.2	{196, 147, 245}	L	49	7 · 7 · 5	7	14	$14^2 + 7^2$ –> $49(2^2 + 1)$	5
13	7	261	6	{180, 189, 261}	U	9	3 · 3 · 29	6	15	$15^2 + 6^2$ –> $9(5^2 + 2^2)$	29
14	7	261	9.1	{189, 180, 261}	L	9	3 · 3 · 29	9	21	$(21^2 + 9^2)/2$ –> $9(7^2 + 3^2)/2$	29
15	8	325	5.1	{125, 300, 325}	U	25	5 · 5 · 13	5	25	$(25^2 + 5^2)/2$ –> $25(5^2 + 1)/2$	13
16	8	325	10	{300, 125, 325}	L	25	5 · 5 · 13	10	15	$15^2 + 10^2$ –> $25(3^2 + 2^2)$	13
17	9	333	3.2	{108, 315, 333}	U	9	3 · 3 · 37	3	18	$18^2 + 3^2$ –> $9(6^2 + 1)$	37
18	9	333	15	{315, 108, 333}	L	9	3 · 3 · 37	15	21	$(21^2 + 15^2)/2$ –> $9(7^2 + 5^2)/2$	37
19	10	369	3.1	{81, 360, 369}	U	9	3 · 3 · 41	3	27	$(27^2 + 3^2)/2$ –> $9(9^2 + 1)/2$	25
20	10	369	12	{360, 81, 369}	L	9	3 · 3 · 41	12	15	$15^2 + 12^2$ –> $9(5^2 + 4^2)$	25
21	11	405	9.1	{243, 324, 405}	U	81	9 · 9 · 5	9	27	$(27^2 + 9^2)/2$ –> $81(3^2 + 1)/2$	5
22	11	405	9.2	{324, 243, 405}	L	81	9 · 9 · 5	9	18	$18^2 + 9^2$ –> $9(6^2 + 1^2)$	5
23	12	425	5.2	{200, 375, 425}	U	25	5 · 5 · 17	5	20	$20^2 + 5^2$ –> $25(4^2 + 1)$	17
24	12	425	15	{375, 200, 425}	L	25	5 · 5 · 17	15	25	$(25^2 + 15^2)/2$ –> $25(5^2 + 3^2)/2$	17
25	13	477	6	{252, 405, 477}	U	9	3 · 3 · 53	6	21	$21^2 + 6^2$ –> $9(7^2 + 2^2)$	53
26	13	477	15	{405, 252, 477}	L	9	3 · 3 · 53	15	27	$(27^2 + 15^2)/2$ –> $9(7^2 + 5^2)/2$	53
27	14	549	3.1	{99, 540, 549}	U	9	3 · 3 · 61	3	33	$(33^2 + 3^2)/2$ –> $9(11^2 + 1)/2$	61
28	14	549	15	{540, 99, 549}	L	9	3 · 3 · 61	15	18	$18^2 + 15^2$ –> $9(6^2 + 5^2)$	61
29	15	585	3.2	{144, 567, 585}	U	9	3 · 3 · 65	3	24	$24^2 + 3^2$ –> $9(8^2 + 1)$	65
30	15	585	9.1	{297, 504, 585}	U	9	3 · 3 · 65	9	33	$(33^2 + 9^2)/2$ –> $9(11^2 + 3^2)/2$	65
31	15	585	12	{504, 297, 585}	L	9	3 · 3 · 65	12	21	$21^2 + 12^2$ –> $9(7^2 + 4^2)$	65
32	15	585	21	{567, 144, 585}	L	9	3 · 3 · 65	21	27	$(27^2 + 21^2)/2$ –> $9(9^2 + 7^2)/2$	65
33	16	605	11	{363, 484, 605}	U	121	11 · 11 · 5	11	33	$(33^2 + 11^2)/2$ –> $121(3^2 + 1)/2$	5
34	16	605	11	{484, 363, 605}	L	121	11 · 11 · 5	11	22	$22^2 + 11^2$ –> $121(2^2 + 1)$	5
35	17	625	5.1	{175, 600, 625}	U	25	5 · 5 · 25	5	35	$(35^2 + 5^2)/2$ –> $25(7^2 + 1)/2$	25
36	17	625	15	{600, 175, 625}	L	25	5 · 5 · 25	15	20	$20^2 + 15^2$ –> $25(4^2 + 3^2)$	25
37	18	637	7.1	{245, 588, 637}	U	49	7 · 7 · 13	7	35	$(35^2 + 7^2)/2$ –> $49(5^2 + 1)/2$	13
38	18	637	14	{588, 245, 637}	L	49	7 · 7 · 13	14	21	$21^2 + 14^2$ –> $49(3^2 + 2^2)$	13
39	19	657	9.2	{432, 495, 657}	U	9	3 · 3 · 73	9	24	$24^2 + 9^2$ –> $9(8^2 + 3^2)$	73
40	19	657	15	{495, 432, 657}	L	9	3 · 3 · 73	15	33	$(33^2 + 15^2)/2$ –> $9(11^2 +5^2)/2$	73

Table FSTS Fermat Systematic Term Sequences From Table RPC1 U Triples

Row	General Terms of FSTS	Span of r(nr) < 3000	Term Count	Primitive Triples
1	$p(\nu_p)$	5 –> 2969	211	1
2	$p(\nu_p) \cdot p(\nu_p)$	5 · 5 = 25 –> 53 · 53 = 2809	7	1
3	$p(1) \cdot p(\nu_p +1)$	5 · 13 = 65 –> 5 · 593 = 2965	50	2
4	$p(2) \cdot p(\nu_p +2)$	13 · 17 = 221 –> 13 · 229 = 2977	20	2
5	$p(3) \cdot p(\nu_p +3)$	17 · 29 = 493 –> 17 · 173 = 2941	15	2
6	$p(4) \cdot p(\nu_p +4)$	29 · 37 = 1073 –> 29 · 101 = 2929	8	2
7	$p(5) \cdot p(\nu_p +5)$	37 · 41 = 1517 –> 37 · 73 = 2701	4	2
8	$p(6) \cdot p(\nu_p +6)$	41 · 53 = 2173 –> 41 · 73 = 2993	3	2
9	$p(\nu_p) \cdot p(\nu_p) \cdot p(\nu_p)$	5 · 5 · 5 = 125 –> 13 · 13 · 13 = 2197	2	1
10	$p(1) \cdot p(2) \cdot p(\nu_p +2)$	5 · 13 · 17 = 1105 –> 5 · 13 · 41 = 2665	4	4
11	$p(1) \cdot p(3) \cdot p(\nu_p +3)$	5 · 17 · 29 = 2465	1	4
12	$p(1) \cdot p(\nu_p +1) \cdot p(\nu_p +1)$	5 · 13 · 13 = 845 –> 5 · 17 · 17 = 1445	2	2
13	$p(1) \cdot p(1) \cdot p(\nu_p + 1)$	5 · 5 · 13 = 325 –> 5 · 5 · 113 = 2825	13	2
14	$p(2) \cdot p(2) \cdot p(\nu_p + 2)$	13 · 13 · 17 = 2873	1	2
15	$p(\nu_p) \cdot p(\nu_p) \cdot p(\nu_p) \cdot p(\nu_p)$	5 · 5 · 5 · 5 = 625	1	1
16	$p(1) \cdot p(1) \cdot p(1) \cdot p(\nu_p +1)$	5 · 5 · 5 · 13 = 1625 –> 5 · 5 · 5 · 17 = 2125	2	2
			344	

Table LS1 Latchkey Search For Fermat Composite 1105 = 5·13·17 Triples

m	s = 2 * m^2	y = r - s	x = sqrt(r^2- y^2)	n = x/(2 * m)	m	s = m^2	y = r - s	X = sqrt(r^2- y^2)	n = x/m
1	2	1103	66.4529909	33.226495	1	1	1104	47	47
2	8	1097	132.7252802	33.18132	3	9	1096	140.7444493	46.914816
3	18	1087	198.6353443	33.105891	5	25	1080	233.7199179	46.743984
4	32	1073	264	33	7	49	1056	325.4059004	46.486557
5	50	1055	328.6335345	32.863353	9	81	1024	415.2697918	46.141088
6	72	1033	392.3467854	32.695565	11	121	984	502.7613748	45.70558
7	98	1007	454.9461507	32.496154	13	169	936	587.3065639	45.177428
8	128	977	516.2325058	32.264532	15	225	880	668.3000823	44.553339
9	162	943	576	32	17	289	816	745.096638	43.829214
10	200	905	634.0346994	31.701735	19	361	744	817	43
11	242	863	690.1130342	31.368774	21	441	664	883.2491155	42.059482
12	288	817	744	31	23	529	576	943	41
13	338	767	795.4470441	30.594117	25	625	480	995.3014619	39.812058
14	392	713	844.1895522	30.149627	27	729	376	1039.061596	38.483763
15	450	655	889.9438185	29.664794	29	841	264	1073	37
16	512	593	932.4033462	29.137605	31	961	144	1095.577017	35.341194
17	578	527	971.2342663	28.565714	33	1089	16	1104.884157	33.481338
18	648	457	1006.06958	27.946377	35	1225	-120	1098.464838	31.38471
19	722	383	1036.501809	27.276363					
20	800	305	1062.073444	26.551836		x	y	r	GCD(x, y, r)
21	882	223	1082.264293	25.768197		47	1104	1105	1
22	968	137	1096.47435	24.919872		817	744	1105	1
23	1058	47	1104	24		943	576	1105	1
24	1152	-47	1104	23		1073	264	1105	1

Table LS2 Latchkey Search For Triples For Composite 3125 = 5 · 5 · 5 · 5 · 5

m	s = 2 * m²	y = r - s	x = sqrt(r² - y²)	n = x/(2 * m)	m	s = m²	y = r - s	x = sqrt(r²- y²)	n = x/m
1	2	3123	111.7855089	55.8927544	1	1	3124	79.0506167	79.0506167
2	8	3117	223.4636436	55.8659109	3	9	3116	237	79
3	18	3107	334.9268577	55.8211429	5	25	3100	394.493346	78.8986692
4	32	3093	446.0672595	55.7584074	7	49	3076	551.2249994	78.7464285
5	50	3075	556.7764363	55.6776436	9	81	3044	706.8868368	78.5429819
6	72	3053	666.9452751	55.5787729	11	121	3004	861.1672311	78.2879301
7	98	3027	776.4637789	55.4616985	13	169	2956	1013.749969	77.9807669
8	128	2997	885.2208764	55.3263048	15	225	2900	1164.313102	77.6208735
9	162	2963	993.1042241	55.1724569	17	289	2836	1312.527714	77.2075126
10	200	2925	1100	55	19	361	2764	1458.056583	76.7398202
11	242	2883	1205.792685	54.8087584	21	441	2684	1600.552717	76.216796
12	288	2837	1310.364835	54.5985348	23	529	2596	1739.657725	75.6372924
13	338	2787	1413.596831	54.3691089	25	625	2500	1875	75
14	392	2733	1515.366622	54.1202365	27	729	2396	2006.192663	74.303432
15	450	2675	1615.549442	53.8516481	29	841	2284	2132.831217	73.545904
16	512	2613	1714.017503	53.563047	31	961	2164	2254.490852	72.7255113
17	578	2547	1810.639666	53.2541078	33	1089	2036	2370.723307	71.8401002
18	648	2477	1905.281082	52.9244745	35	1225	1900	2481.053204	70.8872344
19	722	2403	1997.802793	52.5737577	37	1369	1756	2584.973694	69.8641539
20	800	2325	2088.061302	52.2015325	39	1521	1604	2681.941275	68.767725
21	882	2243	2175.908086	51.8073354	41	1681	1444	2771.369517	67.5943785
22	968	2157	2261.189068	51.3906606	43	1849	1276	2852.621426	66.3400332
23	1058	2067	2343.744013	50.9509568	45	2025	1100	2925	65
24	1152	1973	2423.405868	50.4876222	47	2209	916	2987.736434	63.5688603
25	1250	1875	2500	50	49	2401	724	3039.975164	62.0403095
26	1352	1773	2573.343351	49.4873721	51	2601	524	3080.754615	60.4069532
27	1458	1667	2643.243462	48.948953	53	2809	316	3108.981988	58.6600375
28	1568	1557	2709.49737	48.3838816	55	3025	100	3123.39959	56.7890835
29	1682	1443	2771.89033	47.7912126	57	3249	-124	3122.538871	54.7813837
30	1800	1325	2830.19434	47.1699057					
31	1922	1203	2884.166431	46.5188134					
32	2048	1077	2933.546659	45.8366665		x	y	r	GCD(x, y, r)
33	2178	947	2978.055742	45.1220567		237	3116	3125	1
34	2312	813	3017.392252	44.3734155		1875	2500	3125	625
35	2450	675	3051.22926	43.5889894		2925	1100	3125	25
36	2592	533	3079.210288	42.7668096					
37	2738	387	3100.944372	41.9046537					
38	2888	237	3116	41					
39	3042	83	3123.897566	40.0499688					
40	3200	-75	3124.09987	39.0512484					

Table LS3 Latchkey Search For Triples For Composite 1625 = 5 · 5 · 13

m	s = 2 * m^2	y = r - s	x = sqrt(r^2 - y^2)	n = x/(2 * m)	m	s = m^2	y = r - s	x = sqrt(r^2- y^2)	n = x/m
1	2	1623	80.59776672	40.29888336	1	1	1624	57	57
2	8	1617	161.0465771	40.26164428	3	9	1616	170.7893439	56.92978131
3	18	1607	241.1970149	40.19950248	5	25	1600	283.9454173	56.78908346
4	32	1593	320.8987379	40.11234224	7	49	1576	396.0416645	56.57738064
5	50	1575	400	40	9	81	1544	506.644846	56.29387178
6	72	1553	478.3471543	39.86226286	11	121	1504	615.3121159	55.93746508
7	98	1527	555.7841308	39.69886648	13	169	1456	721.5878325	55.50675635
8	128	1497	632.1518805	39.50949253	15	225	1400	825	55
9	162	1463	707.2877774	39.29376541	17	289	1336	925.0562145	54.41507144
10	200	1425	781.0249676	39.05124838	19	361	1264	1021.238953	53.7494186
11	242	1383	853.1916549	38.78143886	21	441	1184	1113	53
12	288	1337	923.6103074	38.48376281	23	529	1096	1199.753725	52.16320542
13	338	1287	992.0967695	38.15756806	25	625	1000	1280.868846	51.23475383
14	392	1233	1058.459258	37.80211634	27	729	896	1355.658143	50.20956084
15	450	1175	1122.497216	37.41657387	29	841	784	1423.365378	49.08156477
16	512	1113	1184	37	31	961	664	1483.148341	47.84349486
17	578	1047	1242.745348	36.55133376	33	1089	536	1534.056387	46.4865572
18	648	977	1298.497593	36.06937759	35	1225	400	1575	45
19	722	903	1351.005551	35.55277767	37	1369	256	1604.708385	43.37049688
20	800	825	1400	35	39	1521	104	1621.668585	41.58124577
21	882	743	1445.190645	34.40930107	41	1681	-56	1624.03479	39.61060464
22	968	657	1486.262426	33.77869151	43	1849	-224	1609.487185	37.42993454
23	1058	567	1522.870973	33.10589071					
24	1152	473	1554.636935	32.38826948					
25	1250	375	1581.13883	31.6227766		x	y	r	GCD(x, y, r)
26	1352	273	1601.903867	30.8058436		57	1624	1625	1
27	1458	167	1616.395991	29.93325909		825	1400	1625	25
28	1568	57	1624	29		1113	1184	1625	1
29	1682	-57	1624	28		1575	400	1625	25

Table LS4 Latchkey Search For Triples For r = 1185665 = 5 · 13 · 17 · 29 · 37

Row	GTS Table	x	y	r	U/L	m	n	β	n^2+m^2	TSj
1	53.1	81567	1182856	1185665	U	53	1539	1	2371330	TS1
2	64	139136	1177473	1185665	U	64	1087	1	1185665	TS2
3	103.2	223304	1164447	1185665	U	103	1084	1	1185665	TS3
4	183.1	279807	1152176	1185665	U	183	1529	1	2371330	TS1
5	199.1	303873	1146064	1185665	U	199	1527	1	2371330	TS1
6	167.2	359384	1129887	1185665	U	167	1076	1	1185665	TS3
7	191.2	409504	1112703	1185665	U	191	1072	1	1185665	TS3
8	297.1	448767	1097456	1185665	U	297	1511	1	2371330	TS1
9	307.1	463263	1091416	1185665	U	307	1509	1	2371330	TS1
10	236	501736	1074273	1185665	U	236	1063	1	1185665	TS2
11	361.1	540417	1055344	1185665	U	361	1497	1	2371330	TS1
12	281.2	591224	1027743	1185665	U	281	1052	1	1185665	TS3
13	292	612616	1015137	1185665	U	292	1049	1	1185665	TS2
14	449.1	661377	984064	1185665	U	449	1473	1	2371330	TS1
15	359.2	738104	927903	1185665	U	359	1028	1	1185665	TS3
16	543.1	782463	890816	1185665	U	543	1441	1	2371330	TS1
17	449.2	890816	782463	1185665	L	449	992	1	1185665	TS3
18	669.1	927903	738104	1185665	L	669	1387	1	2371330	TS1
19	512	984064	661377	1185665	L	512	961	1	1185665	TS2
20	757.1	1015137	612616	1185665	L	757	1341	1	2371330	TS1
21	771.1	1027743	591224	1185665	L	771	1333	1	2371330	TS1
22	568	1055344	540417	1185665	L	568	929	1	1185665	TS2
23	827.1	1074273	501736	1185665	L	827	1299	1	2371330	TS1
24	601.2	1091416	463263	1185665	L	601	908	1	1185665	TS3
25	607.2	1097456	448767	1185665	L	607	904	1	1185665	TS3
26	881.1	1112703	409504	1185665	L	881	1263	1	2371330	TS1
27	909.1	1129887	359384	1185665	L	909	1243	1	2371330	TS1
28	664	1146064	303873	1185665	L	664	863	1	1185665	TS2
29	673.2	1152176	279807	1185665	L	673	856	1	1185665	TS3
30	981.1	1164447	223304	1185665	L	981	1187	1	2371330	TS1
31	1023.1	1177473	139136	1185665	L	1023	1151	1	2371330	TS1
32	743.2	1182856	81567	1185665	L	743	796	1	1185665	TS3

Table CPL Convergence of Pips to Line $\phi = \pi/4$ (i.e. y = x) Where $|y - x| = \Delta$

ν	m	n	Primitive Triple	ϕ	Table
$\|y–x\| = 1$, $m(\nu+2) = n(\nu)$					
1		3	{3, 4, 5}	53.1301	1.1
2		2	{4, 3, 5}	36.8699	1.2
3		***5***	{20, 21, 29}	46.3972	2
4	3	7	{21, 20, 29}	43.6028	3.1
5	***5***	12	{120, 119, 169}	44.7603	5.2
6	7	17	{119, 120, 169}	45.2397	7.1
7	12	29	{696, 697, 985}	45.0411	12
8	17	41	{697, 696, 985}	44.9589	17.1
9	29	70	{4060, 4059, 5741}	44.9929	29.2
10	41	99	{4059, 4060, 5741}	45.0071	41.1
11	70	169	{23660, 23661, 33461}	45.0012	70
12	99	239	{23661, 23660, 33461}	44.9988	99.1
13	169	408	{137904, 137903, 195025}	44.9998	169.2
14	239	577	{137903, 137904, 195025}	45.0002	239.1
15	408	985	{803760, 803761, 1136689}	45.0000	408
16	577	1393	{803761, 803760, 1136689}	45.0000	577.1
$\|y - x\| = 17$, $m(\nu + 4) = n(\nu)$					
1	1	7	{7, 24, 25}	73.74	1.1
2	2	7	{28, 45, 53}	58.11	2
3	3	4	{24, 7, 25}	16.26	3.2
4	4	11	{88, 105, 137}	50.03	4
5	5	9	{45,28, 53}	31.89	5.1
6	7	***15***	{105, 88, 137}	39.97	7.1
7	7	16	{224, 207, 305}	42.74	7.2
8	9	23	{207, 224, 305}	27.26	9.1
9	11	26	{572, 555, 797}	44.14	11.2
10	***15***	37	{555, 572, 797}	45.86	15.1
11	16	39	{1248, 1265, 1777}	45.39	16
12	23	55	{1265, 1248, 1777}	44.61	23.1
13	26	63	{3276, 3293, 4645}	45.15	26
14	37	89	{3293, 3276, 4645}	44.85	37.1
15	39	94	{7332, 7315, 10357}	44.93	39.2
16	55	133	{7315, 7332, 10357}	45.07	55.1

ν	m	n	Primitive Triple	ϕ	Table
$\|y - x\| = 7$, $m(\nu+4) = n(\nu)$					
1	1	5	{5, 12, 13}	67.38	1.1
2	1	4	{8, 15, 17}	61.93	1.2
3	2	3	{12, 5, 13}	22.62	2
4	3	***5***	{15, 8, 17}	28.07	3.1
5	3	8	{48, 55, 73}	48.89	3.2
6	4	9	{72, 65, 97}	42.08	4
7	5	11	{55, 48, 73}	41.11	5.1
8	***5***	13	{65, 72, 97}	47.92	5.1
9	8	19	{304, 297, 425}	44.33	8
10	9	22	{396, 403, 565}	45.50	9.2
11	11	27	{297, 304, 425}	45.67	11.1
12	13	31	{403, 396, 565}	44.50	13.1
13	19	46	{1748, 1755, 2477}	45.11	19.2
14	22	53	{2332, 2325, 3293}	44.91	22
15	27	65	{1755, 1748, 2477}	44.89	27.1
16	31	75	{2325, 2332, 3293}	45.09	31.1
$\|y – x\| = 23$, $m(\nu + 4) = n(\nu)$					
1	1	3	{12, 35, 37}	71.08	1.2
2	3	11	{33, 56, 65}	59.50	3.1
3	4	7	{56, 33, 65}	30.50	4
4	5	7	{35, 12, 37}	18.90	5.1
5	6	***13***	{156,133, 205}	40.45	6
6	7	19	{133, 156, 205}	49.55	7.1
7	7	18	{252, 275, 373}	47.50	7.2
8	11	25	{275, 252, 373}	42.50	11.1
9	***13***	32	{832, 855, 1193}	45.78	13.2
10	19	45	{855, 832, 1193}	44.22	19.1
11	18	43	{1548, 1525, 2173}	44.57	18
12	25	61	{1525, 1548, 2173}	45.43	25.1
13	32	77	{4928, 4905, 6953}	44.87	32
14	45	109	{4905, 4928, 6953}	45.13	45.1
15	43	104	{8944, 8967, 12665}	45.07	43.2
16	61	147	{8967, 8944, 12665}	44.93	61.1